

**INTRODUCTION to BIOMECHANICS for HUMAN MOTION ANALYSIS,
THIRD EDITION**

SOLUTIONS to ODD-NUMBERED PROBLEMS

by

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INTRODUCTION (p. 12)

Conversion factors are taken from Table 1.3 on page 8.

1.

(a) $350 \times 0.4536 \times 9.81 = 1557 \text{ N}$

(b) $6.50 / 0.4536 = 14.33 \text{ lbs.}$

(c) $168.5 \times 2.54 = 428 \text{ cm}$

(d) $(10 \times 100) / 2.54 = 394 \text{ inches}$

(e) $70.0 \frac{\text{miles}}{\text{hour}} \times \frac{1.609 \text{ km}}{1 \text{ mile}} \times \frac{1 \text{ hour}}{3600 \text{ s}} \times \frac{1000 \text{ m}}{1 \text{ km}} = 31.3 \text{ m/s}$

(f) $80.0 \frac{\text{miles}}{\text{hour}} \times \frac{1.609 \text{ km}}{1 \text{ mile}} = 128.7 \text{ m/s}$

(g) $8.35 \times 12 \times 2.54 = 255 \text{ cm}$

(h) $440 \times 0.9144 = 402 \text{ m}$

(i) $(800 / 0.9144) \times 3 = 2620 \text{ feet}$

(j) $50.0 \times 1.609 = 80.5 \text{ km}$

(k) $25.0 / 1.609 = 15.54 \text{ miles}$

(l) $3.00 \times 9.81 = 29.4 \text{ newtons}$

3.

$$250 \frac{\text{lbs}}{1} \times \frac{0.4536 \text{ kg}}{1 \text{ lbs}} = 113.4 \text{ kg}$$

$$W = mg = 113.4 \times 9.81 = 1112.5 \text{ N}$$

Thus, the 1200 N person weighs more than the 250 lbs. person.

5.

$$45 \text{ ft.} = 45 \times 12 = 540 \text{ in.} = 540 \times 2.54 = 1371.6 \text{ cm} = 13.716 \text{ m}$$

Thus, 13.75 m is longer than 45 feet.

FUNDAMENTAL CONCEPTS (p. 26)

$$\begin{array}{ll} x = r \cos \theta & r = \sqrt{x^2 + y^2} \\ y = r \sin \theta & \theta = \tan^{-1}\left(\frac{y}{x}\right) \end{array}$$

1.

$$\begin{array}{ll} \text{(a)} & r = \sqrt{250^2 + 350^2} = 430 \text{ cm} \\ & \theta = \tan^{-1}(350/250) = 54.5^\circ \end{array} \quad \begin{array}{ll} \text{(b)} & r = \sqrt{2.00^2 + 1.000^2} = 2.24 \text{ kN} \\ & \theta = \tan^{-1}(1.000/2.00) = 26.6^\circ \end{array}$$

$$\begin{array}{ll} \text{(c)} & r = \sqrt{-10.00^2 + -20.0^2} = 22.4 \text{ m/s} \\ & \theta = \tan^{-1}(-20.0/-10.00) = -116.6^\circ \end{array} \quad \begin{array}{ll} \text{(d)} & r = \sqrt{1000^2 + 2000^2} = 2240 \text{ N} \\ & \theta = \tan^{-1}(2000/1000) = 63.4^\circ \end{array}$$

$$\begin{array}{ll} \text{(e)} & r = \sqrt{25.0^2 + (-35.0)^2} = 43.0 \text{ m/s}^2 \\ & \theta = \tan^{-1}(350/250) = -54.5^\circ \end{array} \quad \begin{array}{ll} \text{(f)} & r = \sqrt{-5.00^2 + 15.00^2} = 15.81 \text{ kN} \\ & \theta = \tan^{-1}(15.00/-5.00) = 108.4^\circ \end{array}$$

3.

$$\begin{array}{ll} \text{(a)} & x = 25.0 \cos 25.0^\circ = 22.7 \text{ m/s} \\ & y = 25.0 \sin 25.0^\circ = 10.57 \text{ m/s} \end{array} \quad \begin{array}{ll} \text{(b)} & x = 10.00 \cos 30.0^\circ = 8.66 \text{ N} \\ & y = 10.00 \sin 30.0^\circ = 5.00 \text{ N} \end{array}$$

$$\begin{array}{ll} \text{(c)} & x = 100.0 \cos -45.0^\circ = 70.7 \text{ m/s}^2 \\ & y = 100.0 \sin -45.0^\circ = -70.7 \text{ m/s}^2 \end{array} \quad \begin{array}{ll} \text{(d)} & x = 5.00 \cos(\pi/2) = 0.0 \text{ m} \\ & y = 5.00 \sin(\pi/2) = 5.00 \text{ m} \end{array}$$

$$\begin{array}{ll} \text{(e)} & x = 25.0 \cos 160.0^\circ = -23.5 \text{ N} \\ & y = 25.0 \sin 160.0^\circ = 8.55 \text{ N} \end{array} \quad \begin{array}{ll} \text{(f)} & x = 25.0 \cos(\pi/4) = 17.68 \text{ m/s} \\ & y = 25.0 \sin(\pi/4) = 17.68 \text{ m/s} \end{array}$$

5.

Sum of sides is always greater than length of hypotenuse.

7.

$$\text{(a)} \quad (30.0 - 20.0, 50.0 - 40.0) = (10.00, 10.00) \text{ cm}$$

$$\text{(b)} \quad (400 - 500, 700 - 400) = (-100.0, 300) \text{ N}$$

9.

$$\underline{\mathbf{R}} = (750, 1000) \text{ N} = 1250 \text{ N at } 53.1 \text{ deg}$$

11.

$$r = \sqrt{2.75^2 + 5.25^2} = 5.93 \text{ km}$$

SPATIAL (3D) COORDINATE SYSTEMS (p. 30)

$$P = \sqrt{P_x^2 + P_y^2 + P_z^2}$$

$$\cos \alpha = P_x / P$$

$$\cos \beta = P_y / P$$

$$\cos \gamma = P_z / P$$

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

$$r = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1}(y/x)$$

$$z = z$$

1.

- | | |
|---|--|
| <p>(a) $P = \sqrt{250^2 + 350^2 + 450^2} = 622.5$
 $\cos \alpha = 250/622 = 0.402$
 $\cos \beta = 350/622 = 0.562$
 $\cos \gamma = 450/622 = 0.723$</p> | <p>(b) $P = \sqrt{2^2 + 1^2 + 3^2} = 3.74$
 $\cos \alpha = 2/3.74 = 0.535$
 $\cos \beta = 1/3.74 = 0.267$
 $\cos \gamma = 3/3.74 = 0.802$</p> |
| <p>(c) $P = \sqrt{-10^2 + (-20)^2 + 34^2} = 40.7$
 $\cos \alpha = -10/40.7 = -0.246$
 $\cos \beta = -20/40.7 = -0.492$
 $\cos \gamma = 34/40.7 = 0.836$</p> | <p>(d) $P = \sqrt{1000^2 + 2000^2 + 5000^2} = 5477$
 $\cos \alpha = 1000/5477 = 0.1826$
 $\cos \beta = 2000/5477 = 0.365$
 $\cos \gamma = 5000/5477 = 0.913$</p> |
| <p>(e) $P = \sqrt{25^2 + (-35)^2 + 55^2} = 69.8$
 $\cos \alpha = 25/69.8 = 0.358$
 $\cos \beta = -35/69.8 = -0.501$
 $\cos \gamma = 55/69.8 = 0.788$</p> | <p>(f) $P = \sqrt{-5^2 + 15^2 + 20^2} = 25.5$
 $\cos \alpha = -5/25.5 = -0.1961$
 $\cos \beta = 15/25.5 = 0.588$
 $\cos \gamma = 20/25.5 = 0.785$</p> |
| <p>3. (a) $r = \sqrt{250^2 + 350^2} = 430 \text{ cm}$
 $\theta = \tan^{-1}(350/250) = 54.5^\circ$
 $z = 450 \text{ cm}$</p> | <p>(b) $r = \sqrt{2^2 + 1^2} = 2.24 \text{ m}$
 $\theta = \tan^{-1}(1/2) = 26.6^\circ$
 $z = 3.00 \text{ m}$</p> |
| <p>(c) $r = \sqrt{-10^2 + (-20)^2} = 22.4 \text{ m}$
 $\theta = \tan^{-1}(-20/-10) = 63.4^\circ + 180^\circ$
 $= 243.4^\circ$
 $z = 34.0 \text{ m}$</p> | <p>(d) $r = \sqrt{1000^2 + 2000^2} = 2240 \text{ mm}$
 $\theta = \tan^{-1}(2000/1000) = 63.4^\circ$
 $z = 5000 \text{ mm}$</p> |

$$r = \sqrt{25^2 + (-35)^2} = 43.0 \text{ cm}$$

(e) $\theta = \tan^{-1}(-35/25) = -54.5^\circ$
 $z = 55.0 \text{ cm}$

$$r = \sqrt{-5^2 + 15^2} = 15.81 \text{ cm}$$

(f) $\theta = \tan^{-1}(15/-5) = -71.6^\circ + 180.0^\circ$
 $= 108.4^\circ$
 $z = 20.0 \text{ cm}$

5. (a) $x = 25.0 \cos 35.0^\circ = 20.5 \text{ cm}$
 $y = 25.0 \sin 35.0^\circ = 14.34 \text{ cm}$
 $z = 45.0 \text{ cm}$

(b) $x = 1.100 \cos 35.0^\circ = -0.376 \text{ m}$
 $y = 1.100 \sin 35.0^\circ = 1.034 \text{ m}$
 $z = 0.500 \text{ m}$

(c) $x = 67.5 \cos 45.0^\circ = 47.7 \text{ cm}$
 $y = 67.5 \sin 45.0^\circ = 47.7 \text{ cm}$
 $z = -20.0 \text{ cm}$

(d) $x = 85.0 \cos -120.0^\circ = -42.5 \text{ cm}$
 $y = 85.0 \sin 120.0^\circ = -73.6 \text{ cm}$
 $z = 55.0 \text{ cm}$

RESOLUTION of FORCES into COMPONENTS (p. 37)

$$W = \text{weight} = mg \qquad F_G = \frac{Gm_1m_2}{r^2}$$

$$F_x = F \cos \theta \qquad F_y = F \sin \theta$$

1.

$$W = mg = 50.5 \times 9.81 = 495 \text{ N}$$

3.

$$150 \text{ lbs.} \times \frac{1 \text{ kg}}{2.2045 \text{ lbs}} = 68.2 \text{ kg}$$

$$W = mg = 68.2 \times 9.81 = 669 \text{ N}$$

5.

$$m = \frac{W}{g} = \frac{625}{9.81} = 63.7 \text{ kg}$$

7.

$$(a) \quad \underline{R} = (25 + 10 + 4.53, 30 + 12 - 1.56) = (39.5, 40.4)$$

$$(b) \quad \underline{R} = (3 \times 4.53 - 1.25, 3 \times -1.56 - 5) = (12.34, -9.68)$$

$$(c) \quad \underline{R} = (0 - 10 - 1.25, 10 - 12 - 5) = (-11.25, -7.00)$$

$$(d) \quad \underline{R} = (4.53 + 2(-1.25) - 0, -1.56 + 2(-5) - 10) = (2.03, -21.6)$$

$$(e) \quad \underline{R} = \left[\frac{1}{2}(4.53) + 5.6, \frac{1}{2}(-1.56 - 2) \right] = (7.87, -2.78)$$

$$(f) \quad \underline{R} = \left[\frac{1}{4}(2.53 + 10), \frac{1}{4}(30 + 12) \right] = (8.75, 10.50)$$

$$(g) \quad \underline{R} = (-1.25 - 10 + 0, -5 - 12 + 10) = (-11.25, -7.00)$$

$$(h) \quad \underline{R} = -[(10 - 25), (12 - 30)] = (15.00, 18.00)$$

9.

$$F_{\text{moon}} = \frac{1}{6}(9.81)60.0 = 98.1 \text{ N}$$

$$m_{\text{space}} = 60.0 \text{ kg}$$

$$F_{\text{space}} = 0.00 \text{ N}$$

11.

$$g = \frac{G \times m_{\text{mars}}}{r_{\text{mars}}^2}$$

$$= 6.673 \times 10^{-11} \times \frac{6.419 \times 10^{23}}{(3.396 \times 10^6)^2}$$

$$= 3.71 \text{ m/s}^2$$

MOMENT of FORCE (p. 46–7)

$$M = Fd$$

$$M = rF \sin \theta$$

1.

$$F = \frac{M}{d} = \frac{500}{1.650} = 303 \text{ N}$$

3. (a)

$$R = \Sigma F = -50.0 - 25.0 + 125.0 - 50.0 = 0.00 \text{ N}$$

(b)

$$M_R = \Sigma F_i d_i = 50.0(0.0) - 25.0(0.05) + 125.0(0.15) - 50.0(0.25) \\ = 0 - 1.25 + 18.75 - 12.50 = 5.00 \text{ N.m}$$

(c)

$$\Sigma M_A = 0$$

$$\therefore 125.0(d_C) - 50.0(0.0) - 25.0(0.05) - 50.0(0.25) = 0$$

$$125.0(d_C) = 0.0 + 25.0(0.05) + 50.0(0.25)$$

$$d_C = (1.25 + 12.50)/125.0 = 0.1100$$

The force at C should be moved 11.00 cm to the right of A.

5. (a)

$$\Sigma M = 200(2.50) - 250(2.20) = -50.0$$

Therefore, Cathy will go up.

(b) Jill must move 20.0 cm towards Cathy so the two moments are equal.

(c)

$$\Sigma M = 0$$

$$200(2.50) - 250(2.20) - 150.0(d) = 0$$

$$d = \frac{-50.0}{-150.0} = 0.333$$

Therefore, Ian must sit 33.3 cm from fulcrum on Cathy's side.

7. (a) $M_A = Fd = -300(0.25) = -75.0 \text{ N.m}$ (b) $M_A = F_C \cos \theta (0.20) = -200(\cos 20^\circ)(0.20) = -35.6 \text{ N.m}$

Note, vertical component at C cancels vertical component at D and horizontal component at D has no moment about axis at A.

9. $M = Fd = 56.5(0.650) = 36.7 \text{ N.m}$

LAWS of STATICS (p. 62–4)

$$\begin{array}{ll} \Sigma F_x = 0 & \underline{M} = \underline{r} \times \underline{F} = (r_x F_y - r_y F_x) \underline{k} \\ \Sigma F_y = 0 & \Sigma \underline{F} = \underline{0} \\ \Sigma M_A = 0 & \Sigma \underline{M} = \Sigma(\underline{r} \times \underline{F}) = \underline{0} \end{array}$$

1. (a)

$$\underline{M} = (0.35 \times 80.0 - 0.20 \times 50.0) = 18.00 \underline{k} \text{ N.m}$$

(b)

$$\underline{M} = (0.35 \times 20.0 - 0.20 \times -30.0) = 13.00 \underline{k} \text{ N.m}$$

(c)

$$\underline{M} = [-0.10 \times (80.0 + 20.0) - 0.30 \times (50.0 - 30.0)] = -16.00 \underline{k} \text{ N.m}$$

(d)

$$M = (-0.10 \times 20.0 - 0.30 \times 20.0) = 7.00 \text{ N.m}$$

(e)

$$M = (1.250 \times -50.0 - 2.50 \times 85.0) = -275 \text{ N.m}$$

(f)

$$\underline{M} = (-0.10 \times -50.0 - 0.30 \times 85.0) = -20.5 \underline{k} \text{ N.m}$$

3.

$$\underline{F} = (F \cos \theta, F \sin \theta) = (56.3 \cos 34.3^\circ, 56.3 \sin 34.3^\circ) = (46.51, 31.73)$$

$$\underline{M} = (0.1366 \times 31.73 - 0.205 \times 46.51) = -5.20 \underline{k} \text{ N.m}$$

5. (a)

$$M = (4.00 \times 35.0 - 8.00 \times 25.0) = -60.0 \text{ N.cm} = -0.600 \text{ N.m}$$

(b)

$$M = (-4.00 \times 35.0 - 8.00 \times 25.0) = -340 \text{ N.cm} = -3.40 \text{ N.m}$$

(c)

$$M = (4.00 \times 35.0 - (-8.00) \times 25.0) = 340 \text{ N.cm} = -3.40 \text{ N.m}$$

(d)

$$M = (-4.00 \times 35.0 - (-8.00) \times 25.0) = 60.0 \text{ N.cm} = 0.600 \text{ N.m}$$

7.

$$C = \frac{L \times F_A}{mg} = \frac{2.00 \times 300}{60.0(9.81)} = 1.019 \text{ m} = 101.9 \text{ cm}$$

9.

$$\Sigma F_x = F_{gx} + F_{knee_x} = 0$$

$$\Sigma F_y = F_{gy} + F_{knee_y} - mg = 0$$

$$\Sigma M_{cg} = M_{knee} + (\underline{r}_{knee} \times \underline{F}_{knee}) + (\underline{r}_g \times \underline{F}_g) = 0$$

15.

$$\Sigma F_y = 0: 2F_{cable} - mg = 0$$

$$F_{cable} = \frac{80.0(9.81)}{2} = 392 \text{ N per cable}$$

17. (a)

jacking up a car, prying with a bottle opener, shoveling

(b)

throwing a dart, kicking, jumping

21.

$$\Sigma F_x = 0: -F_{1x} + F_{2x} = 0$$

$$F_{1x} = F_{2x}$$

$$F_{2x} = 200 \cos 30^\circ = 173.2 \text{ N}$$

$$\Sigma F_y = 0: F_{1y} + F_{2y} - W = 0$$

$$F_{1x} = F_{2x} + mg = -200 \sin 30^\circ + 400 = 300 \text{ N}$$

23.

$$\Sigma F_x = 0: F_{knee_x} - F_{load_x} = 0$$

$$F_{knee_x} = F_{load_x}$$

$$F_{knee_x} = 250 \cos 30^\circ = 217 \text{ N}$$

$$\Sigma F_y = 0: F_{knee_y} - F_{load_y} - W = 0$$

$$F_{knee_x} = F_{load_x} + mg = 250 \sin 30^\circ + 40.0 = 165 \text{ N}$$

SPATIAL (3D) LAWS of STATICS (p. 67)

$$\begin{aligned} \Sigma \underline{F} &= \underline{0} & \Sigma \underline{M} &= \Sigma(\underline{r} \times \underline{F}) = \underline{0} \\ \Sigma F_x &= 0 & \Sigma F_y &= 0 & \Sigma F_z &= 0 \\ \Sigma M_{Ax} &= 0 & \Sigma M_{Ay} &= 0 & \Sigma M_{Az} &= 0 \\ \underline{r} \times \underline{F} &= \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix} = (r_y F_z - r_z F_y) \underline{i} - (r_x F_z - r_z F_x) \underline{j} + (r_x F_y - r_y F_x) \underline{k} \end{aligned}$$

1. (a)

$$M_x = 20.0 \times 90.0 - 40.0 \times 80.0 = -1400 \text{ N.cm} = -14.00 \text{ N.m}$$

$$M_y = -(35.0 \times 90.0 - 40.0 \times 50.0) = -1150 \text{ N.cm} = -11.50 \text{ N.m}$$

$$M_z = 35.0 \times 80.0 - 20.0 \times 50.0 = 1800 \text{ N.cm} = 18.00 \text{ N.m}$$

(b)

$$M_x = 20.0 \times 45.0 - 40.0 \times 20.0 = 100.0 \text{ N.cm} = 1.000 \text{ N.m}$$

$$M_y = -(35.0 \times 45.0 - 40.0 \times -30.0) = -2780 \text{ N.cm} = -27.8 \text{ N.m}$$

$$M_z = 35.0 \times 20.0 - 20.0 \times -30.0 = 1300 \text{ N.cm} = 13.00 \text{ N.m}$$

(c)

$$\underline{C} + \underline{D} = (50.0 - 30.0, 80.0 + 20.0, 90.0 + 45.0) = (20.0, 100.0, 135.0)$$

$$M_x = 30.0 \times 135.0 - 0.0 \times 100.0 = 4050 \text{ N.cm} = 40.5 \text{ N.m}$$

$$M_y = -(-10.00 \times 135.0) - 0.0 \times 20.0 = 1350 \text{ N.cm} = 13.50 \text{ N.m}$$

$$M_z = -10.00 \times 100.0 - 30.0 \times 20.0 = -1600 \text{ N.cm} = -16.00 \text{ N.m}$$

(d)

$$5\underline{b} = 5(-10.00, 30.0, 0.0) = (-50.0, 150.0, 0.0)$$

$$M_x = 150.0 \times 90.0 - 0.0 \times 80.0 = 13500 \text{ N.cm} = 135.0 \text{ N.m}$$

$$M_y = -(-50.0 \times 90.0 - 0.0 \times 50.0) = 4500 \text{ N.cm} = 45.0 \text{ N.m}$$

$$M_z = -50.0 \times 80.0 - 150.0 \times 50.0 = -11500 \text{ N.cm} = -115.0 \text{ N.m}$$

(e)

$$\underline{a} + \underline{b} = (35.0 - 10.00, 20.0 + 30.0, 40.0 + 0.0) = (25.0, 50.0, 40.0)$$

$$M_x = 50.0 \times 90.0 - 40.0 \times 80.0 = 1300 \text{ N.cm} = 13.00 \text{ N.m}$$

$$M_y = -(25.0 \times 90.0 - 40.0 \times 50.0) = -250 \text{ N.cm} = -2.50 \text{ N.m}$$

$$M_z = 25.0 \times 80.0 - 50.0 \times 50.0 = -500 \text{ N.cm} = -5.00 \text{ N.m}$$

(f)

$$10\underline{D} = 10(-30.0, 20.0, 45.0) = (-300, 200, 450)$$

$$M_x = 30.0 \times 450 - 0.0 \times 200 = 13500 \text{ N.cm} = 135.0 \text{ N.m}$$

$$M_y = -(-10.00 \times 450) - 0.0 \times -300 = 4500 \text{ N.cm} = 45.0 \text{ N.m}$$

$$M_z = -10.00 \times 200 - 30.0 \times -300 = 7000 \text{ N.cm} = 70.0 \text{ N.m}$$

3.

$$\Sigma \underline{F} = \underline{0} = \underline{F}_1 + \underline{F}_2 - 600 \underline{k} \text{ N}$$

$$\Sigma F_x = 0 = F_{1x} + 150.0 \text{ N}$$

$$\therefore F_{1x} = -150.0 \text{ N}$$

$$\Sigma F_y = 0 = F_{1y} + 65.0 \text{ N}$$

$$\therefore F_{1y} = -65.0 \text{ N}$$

$$\Sigma F_z = 0 = F_{1z} + 350 \text{ N} - 600 \text{ N}$$

$$\therefore F_{1z} = 250 \text{ N}$$

DRY FRICTION (pp. 78–9)

$F_{static} = \mu_{static} F_{normal}$ $F_{kinetic} = \mu_{kinetic} F_{normal}$

1.

$$\Sigma F_n = 0: F_{normal} - mg = 0$$

$$F_{normal} = mg = 50 \times 9.81 = 490.5$$

$$F_{static} = \mu_{static} F_{normal} = 0.95 \times 490.5 = 466 \text{ N}$$

$$F_{kinetic} = \mu_{kinetic} F_{normal} = 0.90 \times 490.5 = 441 \text{ N}$$

3.

$$\Sigma F_n = 0: F_{normal} - mg = 0$$

$$F_{normal} = mg = 35 \times 9.81 = 343.4$$

Since object is in motion :

$$F_{kinetic} = \mu_{kinetic} F_{normal} = 0.90 \times 343.4 = 188.8 \text{ N in negative direction}$$

5.

Since there are only three forces and they must add to zero for statics you can apply the triangle rule :

$$\Sigma \underline{F} = \underline{0}: \underline{F}_{friction} + \underline{F}_{normal} + \underline{W} = \underline{0}$$

be constructing the triangle we see that :

$$F_{friction} = W \sin 15 = 50 \times 9.81 \times 0.25882 = 127.0 \text{ N up the incline}$$

7.

$$\Sigma F_n = 0:$$

$$F_{normal} - mg \cos 12^\circ + F_{applied} \sin 45^\circ = 0$$

$$F_{normal} = 30 \times 9.81 \times \cos 12^\circ - 200 \sin 45^\circ = 146.4$$

$$F_{static} = \mu_{static} F_{normal} = 0.80 \times 146.4 = 117.2$$

Assume $\Sigma F_t = 0$:

$$F_{equilibrium} - mg \sin 12^\circ + F_{applied} \cos 45^\circ = 0$$

$$F_{equilibrium} = 30 \times 9.81 \times \sin 12^\circ - 200 \cos 45^\circ = -80.2$$

since this absolute value is smaller than F_{static} friction is 80.2 N down the incline.

9.

$$\Sigma F_n = 0: F_{normal} - mg - F_{applied} \sin 13^\circ = 0$$

$$F_{normal} = 50 \times 9.81 + 500 \sin 13^\circ = 603.0$$

$$F_{static} = \mu_{static} F_{normal} = 0.80 \times 603 = 482.4 \text{ N}$$

$$\text{Assume } \Sigma F_t = 0: F_{equilibrium} + F_{applied} \cos 13^\circ = 0$$

$$F_{equilibrium} = -500 \cos 13^\circ = -487.2$$

since absolute value is greater than F_{static} the body is moving (friction = $F_{kinetic}$).

11.

$$\Sigma F_n = 0: F_{normal} - W \cos 10^\circ = 0$$

$$F_{normal} = 250 \cos 10^\circ = 246.2$$

$$F_{static} = \mu_{static} F_{normal} = 0.500 \times 246.2 = 123.10 \text{ N}$$

To get the box moving the friction must equal F_{static} .

$$\Sigma F_t = 0: -F_{static} - W \cos 10^\circ + F_{applied} = 0$$

$$F_{applied} = F_{static} + 250 \sin 10^\circ = 123.10 + 43.41 = 166.5 \text{ N}$$

13.

$$F_{static} = \mu_{static} F_{normal} = 0.950(150.0 \times 9.81) = 1398 \text{ N}$$

$$F_{kinetic} = \mu_{kinetic} F_{normal} = 0.900(150.0 \times 9.81) = 1324 \text{ N}$$

15.

$$F_{normal} = 10.00 \times 9.81 = 98.1 \text{ N}$$

$$\mu_{static} = \frac{F_{static}}{F_{normal}} = \frac{25.0}{98.1} = 0.255$$

$$\mu_{kinetic} = \frac{F_{kinetic}}{F_{normal}} = \frac{20.0}{98.1} = 0.204$$

17.

$$F_{normal} = 30.0 \times 9.81 = 294.3 \text{ N}$$

$$\mu_{static} = \frac{F_{static}}{F_{normal}} = \frac{225}{294.3} = 0.765$$

$$\mu_{kinetic} = \frac{F_{kinetic}}{F_{normal}} = \frac{215}{294.3} = 0.731$$

19. No. By definition the coefficient of static friction only occurs at the instant the slipping occurs when the maximum friction also occurs.

LINEAR KINEMATICS (pp. 94–5)

$$v_f = v_i + at$$

$$s_f = s_i + v_i t + \frac{1}{2}at^2$$

$$v_f^2 = v_i^2 + 2a(s_f - s_i)$$

$$s_f = s_i + \frac{1}{2}(v_i + v_f)t$$

1. (a)

$$0 \text{ to } 40 \text{ m: } a = \frac{v_f^2 - v_i^2}{2(s_f - s_i)}$$

$$a = \frac{9^2 - 0}{2(40 - 0)} = \frac{9^2}{2(40)}$$

$$= \frac{81}{80} = 1.0125 \text{ m/s}^2$$

$$40 \text{ to } 70 \text{ m: } a = 0 \text{ m/s}^2$$

$$70 \text{ to } 100 \text{ m: } s_f = s_i + vt + \frac{1}{2}at^2$$

$$100 = 70 + 9(4) + \frac{1}{2}a(4)^2$$

$$a = 2 \frac{(30 - 36)}{16} = -0.750 \text{ m/s}^2$$

(b)

$$t_{0-40} = \frac{v_f - v_i}{a} = \frac{9 - 0}{1.0125} = 8.89 \text{ s}$$

$$t_{40-70} = \frac{70 - 40}{9} = 3.33 \text{ s}$$

$$t_{70-100} = 4.00 \text{ s}$$

$$t_{\text{total}} = 8.89 + 3.33 + 4 = 16.22 \text{ s}$$

(c)

$$s_f = 0 + 0 + \frac{1}{2} \left(\frac{81}{80} \right) 5^2 = 12.66 \text{ m}$$

(d)

$$v_{100} = 9 - \left(\frac{3}{4} \right) 4 = 6.00 \text{ m/s}$$

3. (a)

$$v_f^2 = \pm \sqrt{3.5^2 + 2(0.005)(25 - 0)}$$

$$= \pm 3.536 \text{ m/s}$$

$$t = \frac{v_f - v_i}{a} = \frac{+3.536 - 3.50}{0.005} = 7.11 \text{ s}$$

(b)

$$s_f = s_i + v_i t + \frac{1}{2}at^2$$

$$= 0 + 3.5 \times 10^2 + \frac{1}{2}(0.005)10^2$$

$$= 35 + 0.25 = 35.3 \text{ m}$$

5. (a)

$$t = \frac{v_f - v_i}{a} = \frac{0 - 2.20}{-0.300} = 7.33 \text{ s}$$

(b)

$$s_f = s_i + v_i(7.333) + \frac{1}{2}at^2$$

$$= 0 + 2.20(7.333) +$$

$$\frac{1}{2}(-0.3)(7.33)^2 = 8.07 \text{ m}$$

7.

$$v = 150 \text{ km/h} \div 3.6 = 41.67 \text{ m/s}$$

1st catcher :

$$a = \frac{v_f^2 - v_i^2}{2(s_f - s_i)} = \frac{0 - 41.67^2}{2(0.300 - 0)}$$

$$= -2894$$

$$t = \frac{v_f - v_i}{a} = \frac{0 - 41.67}{-2894}$$

$$= 0.01440 \text{ s} = 14.40 \text{ ms}$$

2nd catcher :

$$a = \frac{v_f^2 - v_i^2}{2(s_f - s_i)} = \frac{0 - 41.67^2}{2(0.500 - 0)}$$

$$= -1736$$

$$t = \frac{v_f - v_i}{a} = \frac{0 - 41.67}{-1736}$$

$$= 0.0240 \text{ s} = 24.0 \text{ ms}$$

The difference in the accelerations is 1158 m/s^2 . The difference in the times is 9.60 ms .

9.

$$s = \sqrt{12.00^2 + \left(\frac{7.32}{2}\right)^2} = 12.546 \text{ m}$$

$$v = s/t = 12.546/1.500 = 8.36 \text{ m/s}$$

Ball speed must be greater than 8.36 m/s.

11. (a)

$$v = \frac{s}{t} = \frac{100 \text{ m}}{4 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ s}} = 0.417 \text{ m/s}$$

(b)

$$s_{\text{down river}} = v_{\text{current}}t = 4.00 \frac{\text{km}}{\text{h}} \times \frac{1 \text{ h}}{60 \text{ min}} \times \frac{1000 \text{ m}}{1 \text{ km}} \times 4.00 \text{ min} = 266.7 \text{ m}$$

$$s_{\text{total}} = \sqrt{266.7^2 + 100.0^2} = 285 \text{ m}$$

(c)

$$v = s_{\text{total}}/t = 284.8/(4.00 \times 60.0) = 1.187 \text{ m/s}$$

13.

$$0 = v_i^2 + 2a(s_f - s_i)$$

$$v_i^2 = -2a(s_f - s_i) = -2(-2.00)(6.00 - 0)$$

$$v_i = \pm\sqrt{24.0} = \pm 4.90$$

The initial velocity must be 4.90 m/s.

PROJECTILE MOTION (pp. 101–2)

$$\begin{aligned}
 v_{fy} &= v_{iy} - gt \\
 s_{fy} &= s_{iy} - gt \\
 v_{fy}^2 &= v_{iy}^2 - 2g(s_{fy} - s_{iy}) \\
 s_{fy} &= s_{iy} + \frac{1}{2}(v_{iy} + v_{fy})t
 \end{aligned}$$

1. (a)

$$90 \frac{\text{km}}{\text{h}} \times \frac{1000 \text{ m}}{1 \text{ km}} \times \frac{1 \text{ h}}{3600 \text{ s}} = 25.0 \text{ m/s}$$

$$v_x = v \cos \theta = 25 \cos 5^\circ = 24.90 \text{ m/s}$$

$$v_y = v \sin \theta = 25 \sin 5^\circ = 2.179 \text{ m/s}$$

(b)

$$s_{fx} = s_{ix} - v_x t$$

$$t = \frac{10 - 0}{24.9} = 0.402 \text{ s}$$

(c)

$$v_{fy}^2 = v_{iy}^2 - 2g(s_{fy} - s_{iy})$$

$$s_{fy} = \frac{v_{fy}^2 - v_{iy}^2}{-2g} + s_{iy}$$

$$= \frac{0 - (2.179)^2}{-2(9.81)} + 0 = 0.242 \text{ m} = 24.2 \text{ cm}$$

(d)

$$v_x = \frac{10 - 0}{0.350} = 28.6 \text{ m/s}$$

3. (a)

$$v_{fy}^2 = v_{iy}^2 - 2g(s_{fy} - s_{iy})$$

$$v_{fy} = \pm \sqrt{0.2(9.81)(-20 - 0)}$$

$$= \pm \sqrt{392.4} = \pm 19.81 \text{ m/s}$$

$$v_f = -19.81 \text{ m/s}^2$$

(b)

$$t = \frac{v_f - v_i}{-g} = \frac{-19.81 - 0}{-9.81} = 2.02 \text{ s}$$

(c)

$$s_{ix} = s_{ix} - v_x t = 0 + 2(2.02) = 4.04 \text{ m}$$

5. (a)

$$\begin{aligned}
 s_{fy} &= s_{iy} + \frac{v_{fy}^2 - v_{iy}^2}{-2g} \\
 &= 0 + \frac{0 + v_{iy}^2}{+2g} = \frac{6.00^2}{2(9.81)} = 1.835 \text{ m}
 \end{aligned}$$

(b)

$$t = \frac{v_f - v_i}{-g}$$

$$v_f^2 = v_i^2 - 2g(s_f - s_i)$$

$$= 6^2 - 2(9.81)(0 - 10)$$

$$v_f = \pm \sqrt{232.2} = \pm 15.24 \text{ m/s}$$

select the negative velocity

$$t = \frac{-15.24 - 6}{-9.81} = 2.16 \text{ s}$$

(c)

$$s_x = v_x t = 0.500(2.16) = 1.082 \text{ m}$$

7.

θ	v_x	v_y	y_{max}	x_{max}	time
30°	8.66	5.00	1.270	8.83	1.019
45°	7.07	7.07	2.55	10.19	1.442
60°	5.00	8.66	3.82	8.83	1.765

9.

$$v_{fy}^2 = 0 - 2g(1.0 - 1.5) = 9.81$$

$$v_{fy} = \pm \sqrt{9.81} = -3.13$$

$$t = \frac{v_f - v_i}{-g} = \frac{-3.13 - 0}{-9.81} = 0.319 \text{ s}$$

$$v_x = \frac{10}{0.319} = 31.3 \text{ m/s}$$

11.

$$v_f^2 = 0 - 2(9.81)(-0.795)$$

$$v_f = -3.94 \text{ m/s}$$

$$t = \frac{v_f}{g} = 0.4026$$

$$v_{fps} = \frac{12}{0.4206} = 29.8 \text{ fps}$$

ANGULAR KINEMATICS (p. 106)

$$\omega_f = \omega_i + \alpha t$$

$$\theta_f = \theta_i + \omega_i t + \frac{1}{2} \alpha t^2$$

$$\omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i)$$

$$\theta_f = \theta_i + \frac{1}{2}(\omega_i + \omega_f)t$$

1. (a)

$$\alpha = \frac{\omega_f - \omega_i}{t} = \frac{5 - 2}{2} = 1.500 \text{ r/s}^2$$

$$= 3\pi \text{ rad/s}^2 = 9.42 \text{ rad/s}^2$$

(b)

$$\theta_f = \theta_i + \omega_i t + \frac{1}{2} \alpha t^2$$

$$= 0 + 2(2) + \frac{1}{2}(1.5)2^2 = 7.00 \text{ revolutions}$$

3.

$$\theta = 0 + 0 \frac{1}{2} (0.35) 36^2$$

$$= 227 \text{ revolutions}$$

5.

$$\omega_f = \omega_i + \alpha t$$

$$= 6 + 0.750(5) = 9.75 \text{ rad/s}$$

7.

$$\omega = \frac{3r}{0.89} = \frac{6\pi}{0.89} = 21.2 \text{ rad/s}$$

9.

$$\omega_{final} = \omega_{initial} + \alpha t = 5.60 + (-0.200)3.00 = 5.00 \text{ rad/s}$$

11.

$$t = \frac{\omega_{final} - \omega_{initial}}{\alpha} = \frac{0 - 20.0}{-1.500} = 13.33 \text{ s}$$

RELATIONSHIP between LINEAR and ANGULAR MEASURES (pp. 111–2)

$$v_t = r\omega$$

$$a_t = r\alpha$$

$$a_r = r\omega^2 = \frac{v_t^2}{r}$$

$$a = \sqrt{a_r^2 + a_t^2}$$

1. (a)

$$v_t = r\omega = 0.75(15) = 11.25 \text{ m/s}$$

(b)

$$a_t = r\alpha = 0.75(150) = 117.5$$

$$a_r = r\omega^2 = 0.75(15)^2 = 168.75$$

$$a = \sqrt{a_t^2 + a_r^2} = \sqrt{117.5^2 + 168.75^2}$$

$$= 203 \text{ m/s}^2$$

3. (a)

$$v_t = r\omega = 0.900 \left(573 \frac{\text{deg}}{\text{s}} \right) \times \frac{\pi \text{ rad}}{180 \text{ deg}}$$

$$= 9.00 \text{ m/s}$$

(b)

$$\alpha = \frac{\omega_f - \omega_i}{t} = \frac{10 - 0}{1.5} = 6.67 \text{ rad/s}^2$$

(c)

$$a_t = r\alpha = 0.9(10) = 9.00$$

$$a_r = r\omega^2 = 0.9(10)^2 = 90.0$$

$$a = \sqrt{a_t^2 + a_r^2} = \sqrt{9.00^2 + 90.0^2}$$

$$= 90.4 \text{ m/s}^2$$

5.

$$v_t = r\omega$$

$$= 0.75(10.00)$$

$$= 7.50 \text{ m/s}$$

$$a_r = r\omega^2 = 0.75(10)^2 = 75.0 \text{ m/s}^2$$

$$a_t = r\alpha = 0.75(2.00) = 1.500 \text{ m/s}^2$$

$$a = \sqrt{a_t^2 + a_r^2} = \sqrt{75^2 + 1.5^2}$$

$$= 75.0 \text{ m/s}^2$$

7. (a)

$$v_{feet} = r\omega = 2.30(10) = 23.0 \text{ m/s}$$

$$v_{cg} = r\omega = 1.30(10) = 13.00 \text{ m/s}$$

(b)

$$\omega = \frac{v}{r} = \frac{10 - 0}{2} = 5.00 \text{ rad/s}$$

(c)

$$s_{feet} = 2\pi r = 2\pi(2.30) = 14.45 \text{ m}$$

$$s_{cg} = 2\pi r = 2\pi(1.3) = 8.16 \text{ m}$$

$$= 14.45 - 8.16 = 6.29 \text{ m}$$

(d)

$$a_{r\text{ feet}} = r\omega^2 = 2.30(10.00)^2$$

$$= 230 \text{ m/s}^2$$

9. (a)

$$r_{total} = 1.00 + .70 = 1.700 \text{ m}$$

$$v_t = r\omega = 1.70(8.75) = 14.875 \text{ m/s}$$

$$v = r\omega; \quad r = \frac{v}{\omega},$$

$$1.70 + r = \frac{v_t + 1.5}{\omega}$$

$$r = \frac{16.375}{8.75} - 1.7 = 0.1714 \text{ m}$$

(b)

$$a_r = r\omega^2 = 0.1714(8.75)^2$$

$$= 13.12 \text{ m/s}^2$$

11.

$$\omega = 65 \frac{\text{r}}{\text{min}} \times \frac{2\pi \text{ rad}}{1 \text{ r}} \times \frac{1 \text{ min}}{60 \text{ s}}$$

$$= 6.81 \text{ rad/s}$$

$$v_{rim} = r\omega = 0.330(6.81) = 2.25 \text{ m/s}$$

$$v_{pushwheel} = r\omega = 0.100(6.81)$$

$$= 0.681 \text{ m/s}$$

LAW of ACCELERATION (p. 117)

$$\begin{aligned}\Sigma \underline{F} &= m \underline{a} \\ \Sigma F_x &= ma_x \\ \Sigma F_y &= ma_y\end{aligned}$$

1.

$$a = \left(\frac{v_f^2 - v_i^2}{2(x_f - x_i)} \right) = \left(\frac{0 - 9^2}{2(30 - 0)} \right) = \frac{-81}{60} = -1.350 \text{ m/s}^2$$

$$F = ma = 60.0(-1.350) = -81.0 \text{ N}$$

3.

$$\Sigma F_y = ma_y = F_{\text{lifter}} - mg$$

$$a_y = \left(\frac{v_{yf} - v_{yi}}{t} \right) = \frac{2 - 0}{1} = 2.00$$

$$F_{\text{lifter}} = ma_y + mg = 40(2) + 40(9.81) = 472 \text{ N}$$

5.

$$F_{\text{normal}} = mg = 70(9.81) = 686.7$$

$$F_{\text{kinetic}} = F_{\text{normal}} \mu_{\text{kinetic}} = 686.7 \times 0.6 = 412 \text{ N}$$

$$\Sigma F_x = ma_x = -F_{\text{kinetic}}$$

$$a_x = \frac{-F_{\text{kinetic}}}{70} = -5.886 \text{ m/s}^2$$

$$\text{Thus, } x_f = x_i + \left(\frac{v_{fx}^2 - v_{ix}^2}{2a_x} \right) = 0 + \left(\frac{0 - 10^2}{2(-5.886)} \right) = 8.49 \text{ m}$$

7.

$$\Sigma F = ma = 1000 = 900a$$

$$a = 1000/900 = 1.111 \text{ m/s}^2$$

$$t = \frac{v_f - v_i}{a} = \frac{3 - 0}{1.111} = 2.70 \text{ s}$$

9.

$$a = \frac{v_f - v_i}{t} = \frac{0 - 20}{0.3} = -66.67 \text{ m/s}^2$$

$$F = ma = 0.180(-66.67) = -12.00 \text{ N}$$

11.

$$\Sigma F_x = ma_x = 35 + 58 - 52 = 41.0 \text{ N}$$

$$\Sigma F_y = ma_y = 25 - 20 + 23 - mg = 28 - 196.2 = -168.2 \text{ N}$$

$$a_x = 41/20 = 2.05 \text{ m/s}^2$$

$$a_y = -168.2/20 = -8.41 \text{ m/s}^2$$

MOMENT of FORCE (p. 122)

$$M_a = [\underline{r} \times \underline{F}]_z = (r_x F_y - r_y F_x)$$

$$M_R = \Sigma M_a = I_a \alpha$$

$$M = F d$$

1.

$$\alpha = \frac{M}{I} = 35/2 = 17.50 \text{ rad/s}^2$$

3.

$$M = Fd = 150(0.35) = 52.5 \text{ N.m}$$

$$\alpha = M / I = 2.10 \text{ rad/s}^2$$

5.

$$\alpha = \frac{\omega_f - \omega_i}{t} = \frac{2.35 - 4.45}{5.00} = -0.420$$

$$M = I\alpha = 32.0(-0.42) = -13.44 \text{ N.m}$$

7.

$$M = Fd = 400 \times 0.35 = 140.0 \text{ N.m}$$

9.

$$M = Fd = 500 \times 0.40 = 200 \text{ N.m}$$

$$I = M / \alpha = 200 / 20 = 10.00 \text{ kg.m}^2$$

MOMENT of INERTIA (p. 129)

$$\begin{aligned}
 K_{cg} &= k_{cg} / L \\
 I_{cg} &= mk_{cg}^2 \\
 I_{axis} &= I_{cg} + mr^2
 \end{aligned}$$

1.

$$k_{cg} = \sqrt{\frac{I_{cg}}{m}} = \sqrt{\frac{0.5}{8}} = 0.250 \text{ m}$$

$$\begin{aligned}
 I_{axis} &= I_{cg} + mr^2 = 0.5 + 8(0.25)^2 \\
 &= 1.000 \text{ kg.m}^2
 \end{aligned}$$

3.

$$k = KL = 0.60(0.95) = 0.57 \text{ m}$$

$$I_{cg} = mk^2 = 12.00(0.57)^2 = 3.90 \text{ kg.m}^2$$

$$\begin{aligned}
 I_{hip} &= I_{cg} + mr^2 \\
 &= 3.90 + 12.00(0.50)^2 = 6.90 \text{ kg.m}^2
 \end{aligned}$$

5. (a)

$$\begin{aligned}
 I_{bar} &= I_{cg} + mr^2 = 15.80 + 80.0(1.450)^2 \\
 &= 15.8 + 168.2 = 184.0 \text{ kg.m}^2
 \end{aligned}$$

(b)

$$k = \sqrt{\frac{15.80}{80.0}} = \sqrt{0.1975} = 0.444 \text{ m}$$

7.

$$M = Fd = 300(0.15) = 45.0 \text{ N.m}$$

$$\alpha = (\omega_f - \omega_i) / t = 20 / 1 = 20.0 \text{ rad/s}^2$$

$$I = M / \alpha = 45 / 20 = 2.25 \text{ kg.m}^2$$

$$k = \sqrt{I / m} = \sqrt{2.25 / 90.0} = 0.1581 \text{ m}$$

9.

$$\begin{aligned}
 I_{proximal} &= I_{cg} + mr^2 = 0.1489 + 7.05(0.1925)^2 \\
 &= 0.410 \text{ kg.m}^2
 \end{aligned}$$

$$k_{proximal} = \sqrt{I_{proximal} / m} = \sqrt{0.453 / 7.05} = 0.241 \text{ m} = 2.41 \text{ cm}$$

REACTION FORCES (p. 136)

$$\underline{F = -R}$$

$$F_{\text{centripetal}} = mr\omega^2 = mv^2 / r$$

1.

$$\Sigma F_x = F_{gx} = ma_x$$

$$a_x = F_{gx} / m = 227 / 53.8 = 4.22 \text{ m/s}^2$$

$$\Sigma F_y = F_{gy} - mg = ma_y$$

$$a_y = (1345 - 53.8 \times 9.81) / 53.8 = 15.19 \text{ m/s}^2$$

3.

$$s = r\theta$$

$$r = s / \theta = 100 / \pi = 31.83$$

$$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right) = \frac{11^2}{31.83 \times 9.81}$$

$$= \tan^{-1}(0.3875) = 21.2 \text{ deg}$$

5.

$$mg = m \frac{v^2}{r}$$

$$r = \frac{v^2}{g} = \frac{250^2}{9.81} = 6371 \text{ m} = 6.37 \text{ km}$$

7.

$$\Sigma F_y = ma_y = F_{gy} - mg$$

$$F_{gy} = ma_y + mg = 65.0(0.75 + 9.81)$$

$$= 686 \text{ N}$$

9.

$$ma_r = -mv_t^2 / r = -mr\omega^2$$

$$= -3.50(2.00)^2 = -175.0 \text{ N}$$

11.

$$F = ma = 75(20 \times 9.81) = 14\,720 \text{ N}$$

LINEAR IMPULSE and MOMENTUM (p. 150)

$$\begin{aligned} \text{momentum} &= p = mv \\ \text{impulse} &= \int F dt = \bar{F}t \\ \int F dt &= mv_f - mv_i \end{aligned}$$

1. $\bar{F} = m\bar{a} = 60.0 \left(\frac{0 - 12.00}{1.100} \right) = 60(-10.909) = -655 \text{ N}$
3. Impulse = $Ft = mv_f - mv_i = 55.0(2.50) - 0 = 137.5 \text{ N}\cdot\text{s}$
5. $mv_{fx} = mv_{ix} + \int F_x dt = 0 + 200$
 $v_{fx} = 200 / 70.0 = 2.86 \text{ m/s}$
 $mv_{fy} = mv_{iy} + \int F_y dt - Wt$
 $= 0 + 1200 - 70(9.81)1.2 = 375.86$
 $v_{fy} = 375.86 / 70.0 = 5.37 \text{ m/s}$
7. $v_{fy}^2 = v_{iy}^2 - 2g(y_f - y_i)$
 $= 0 - 2(9.81)(-1.350 - 0) = 26.487$
 $v_{fy} = \sqrt{26.476} = -5.1247 \text{ m/s}$
 $a_{\text{landing}} = \frac{v_f - v_i}{t} = \frac{0 - (-5.127)}{0.400} = 12.87 \text{ m/s}^2$
 $F_{\text{landing}} = ma_{\text{landing}} = 60.0(12.87) = 772 \text{ N}$
9. $\bar{F}t = mv_f - mv_i$
 $\bar{F} = \frac{-mv_i}{t} = \frac{-0.005(400)}{0.050} = -40.0 \text{ N}$
11. $v_f = v_i + \frac{\bar{F}t}{m} = 0 + 24.0(1) / 4 = 6.00 \text{ m/s}$

ANGULAR IMPULSE and MOMENTUM (p. 155)

$$\begin{aligned} \text{angular momentum} &= L = I\omega \\ \text{angular impulse} &= \int Mdt = \overline{M}t \\ \int Mdt &= I\omega_f - I\omega_i \end{aligned}$$

1. $L = I\omega = 5.00(5.52)$
 $= 27.6 \text{ kg}\cdot\text{m}^2/\text{s}$

3. $L = I\omega = 5.65 \left(2.25 \frac{r}{s} \times \frac{2\pi \text{ rad}}{1 r} \right)$
 $= 5.65(14.137) = 79.9 \text{ kg}\cdot\text{m}^2/\text{s}$

5. $\overline{M}t = (\overline{F}d)t = 250(1.350)0.500$
 $= 168.8 \text{ N}\cdot\text{m}\cdot\text{s}$

7. $\overline{M}t = (\overline{F}d)t = 68.0(0.320)5$
 $= 108.8 \text{ N}\cdot\text{m}\cdot\text{s}$

9. $k = Kl = 0.326(1.235) = 0.4026 \text{ m}$
 $I = mk^2 = 11.50(0.4026)^2 = 1.8641 \text{ kg}\cdot\text{m}^2$
 $L = I\omega = 1.8641(2.55) = 4.75 \text{ kg}\cdot\text{m}^2/\text{s}$

CONSERVATION of MOMENTUM (p. 163)

$$p_f = p_i = mv = \text{constant}$$

$$L_f = L_i = I\omega = \text{constant}$$

1. (a)

$$L = I\omega = 2(20) = 40.0 \text{ kg}\cdot\text{m}^2/\text{s}$$

(b)

$$I_{top}\omega_{top} = L_{start} = L_{top} = L_{land} = 40.0 \text{ kg}\cdot\text{m}^2/\text{s}$$

$$I_{top} = L / 30.0 = 1.333 \text{ kg}\cdot\text{m}^2$$

$$I_{land} = L / 26.6 = 1.504 \text{ kg}\cdot\text{m}^2$$

3. (a)

$$L = I\omega$$

$$8.25 = 0.430\omega$$

$$\omega = 8.25 / 0.43 = 19.19 \text{ rad/s}$$

(b)

$$M = I\alpha = 0.43 \left(\frac{19.19 - 0}{0.5} \right) = 16.50 \text{ N}\cdot\text{m}$$

5.

$$\omega = L / I = 8.95 / 16.26 = 0.5504$$

$$\omega = \frac{\theta_f - \theta_i}{t}$$

$$t = \frac{\theta_f - 0}{\omega} = \frac{2p}{0.5504} = 11.42 \text{ s}$$

7.

$$I = L / \omega = 35.6 / 3.25 = 10.95 \text{ kg}\cdot\text{m}^2$$

WORK-ENERGY THEOREM (p. 174)

$$\text{work} = W = \Delta E = E_f - E_i$$

$$\text{energy} = E = mgy + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

1.

$$E = mgy + \frac{1}{2}mv^2 = 70(9.81)1.150 + \frac{1}{2}(70)12.00^2$$

$$= 789.7 + 5040 = 5830 \text{ joules} = 5.83 \text{ kJ}$$

3.

$$E = \frac{1}{2}mv^2 = \frac{1}{2}(60.0)2.00^2 = 120.0 \text{ J}$$

5.

$$E = \frac{1}{2}I\omega^2 = \frac{1}{2}(2.00) \left(300 \frac{r}{\text{min}} \times \frac{2\pi}{1r} \times \frac{1 \text{ min}}{60s} \right)^2$$

$$= \frac{1}{2}(2.00)31.416^2 = 987 \text{ J}$$

7.

$$y = v \sin 20^\circ t = 6.00 \sin 20^\circ (60.0 \text{ s}) = 123.1 \text{ m}$$

$$W = E_f - E_i = mgy_f - mgy_i$$

$$= 50(9.81)123.1 - 0 = 490.5(123.1) = 60.4 \text{ kJ}$$

9.

$$W = E_f - E_i = 0 - E_i = 0 - \frac{1}{2}mv^2 = -\frac{1}{2}(25.0)5^2 = -313 \text{ J}$$

11. (a)

$$a = \frac{v_f - v_i}{t} = \frac{0 - 3}{4} = -0.750$$

$$s_f = s_i + \frac{v_f^2 - v_i^2}{2a} = \frac{0 - 3^2}{2(-0.75)} = \frac{-9.0}{-1.50} = 6.00 \text{ m}$$

(b)

$$\bar{F} = m\bar{a} = 20.0(-0.75) = -15.00 \text{ N}$$

(c)

$$W = Fs = -15.00(6.00) = -90.0 \text{ J}$$

WORK of a FORCE or MOMENT of FORCE (p. 180–1)

$$W_{force} = Fs \cos \phi$$

$$W_{force} = \underline{F} \cdot \underline{s} = F_x s_x + F_y s_y$$

$$W_{moment} = M \theta = (Fr \sin \phi) \theta$$

1. (a) $W = Fs = 358 \times 0.517 = 185.1 \text{ J}$
- (b) $W = F_x s_x + F_y s_y$
 $= 25.3 \times 1.325 + 63.2 \times 2.92$
 $= 218 \text{ J}$
3. $W_{force} = Fs \cos \phi$
 $= 35.0 (23.0) \cos 30^\circ = 697 \text{ J}$
5. $F_{normal} = mg = 25.0(9.81)$
 $= 245.25 \text{ N}$
 $F_{kinetic} = F_{normal} \mu_{kinetic}$
 $= 245.25 \times 0.800 = 196.2 \text{ N}$
 $W_{\mu=0.8} = F_{kinetic} s = 196.2 \times 10.00$
 $= 1962 \text{ J}$
 $W_{\mu=0.2} = 245.25 \times 0.2 \times 10.00$
 $= 491 \text{ J}$
7. $W = Fs = (Lg)s$
 $= 20(9.81)10(6.00)$
 $= 196.2(60.0) = 11772 \text{ J}$
 $= 11.77 \text{ kJ}$
9. $W = Fs = (Lg)s$
 $= 3.50(9.81)4000$
 $= 137340 \text{ J} = 137.3 \text{ kJ}$
11. $W = mgy = 60(9.81)0.350$
 $= 206 \text{ J}$
13. (a) $W_{total} = mgy = 300(9.81)0.1500$
 $= 441 \text{ J}$
- (b) $W_{moment} = M\theta = (Fd)\theta$
 $= (90.0 \times 0.600)(1 \text{ rad}) = 54.0 \text{ J}$
- (c) $n = W_{total} / W_{moment}$
 $= \frac{441}{54} = 8 \text{ and } 1/6 = 8.17 \text{ cycles}$
15. (a) $W = E_f - E_i = 0 - \frac{1}{2} I \omega^2$
 $= -\frac{1}{2} (0.450) 20.0^2 = -90.0 \text{ J}$
- (b) $F_{friction} = F_{normal} \mu_{kinetic}$
 $= -12.50(0.800) = -10.00 \text{ N}$
 $\alpha = \frac{M}{I} = \frac{Fd}{I} = \frac{10.00 \times 0.300}{0.450}$
 $= -6.667 \text{ rad/s}^2$
 $t = \frac{\omega_f - \omega_i}{\alpha} = \frac{0 - 20.0}{-6.667} = 3.00 \text{ s}$

POWER (p. 185)

$$P = W / t = \Delta E / t$$

$$P_{force} = Fv \cos\phi$$

$$P_{force} = \underline{F} \cdot \underline{v} = F_x v_x + F_y v_y$$

$$P_{moment} = M\omega$$

1.

$$s = 2 \times 70.0 \times 6.00 = 840 \text{ metres}$$

$$W_L = 10.00 \times 9.81 = 98.1 \text{ joules}$$

$$P = W_L s / t = 98.1(840) / (2 \times 60) \\ = 687 \text{ W}$$

3.

$$P = (W_L g) s / t = (3.5 \times 9.81) \times 4000 / (6 \times 60) = 382 \text{ W}$$

5.

$$P = 0$$

An isometric contraction does no mechanical work.

7.

$$\omega = 200 \frac{\text{deg}}{\text{s}} \times \frac{2\pi \text{ rad}}{360 \text{ deg}} = 3.49$$

$$P = M\omega = 50.0 \times 3.39 = 174.5 \text{ W}$$

9.

$$\omega = 3 \frac{r}{s} \times \frac{2\pi \text{ rad}}{1r} = 18.85 \text{ rad/s}$$

$$P = M\omega = 55.0 \times 18.85 = 1037 \text{ W}$$

11.

$$P = Fv = 225 \times 0.555 = 124.9 \text{ W}$$

$$W = Pt = 124.9 \times 3.50 = 437 \text{ J}$$

CONSERVATION of MECHANICAL ENERGY (p. 190)

$$E_f = E_i = \text{constant}$$

1.

$$W_G = mgy = 75.0 \times 9.81 \times 3.00 = 2210 \text{ J}$$

$$\frac{1}{2}mv^2 = mgy$$

$$v = \sqrt{2gy} = \sqrt{2(9.81)3} = 7.67 \text{ m/s}$$

3.

$$E_f = E_i$$

$$\frac{1}{2}mv_f^2 + mgy_f = \frac{1}{2}mv_i^2 + mgy_i$$

$$\frac{1}{2}mv_f^2 + 0 = \frac{1}{2}(65.0)4.15^2 + 65.0(9.81)10.00$$

$$= 559.7 + 6376.5 = 6936 \text{ J}$$

$$v_f = \sqrt{\frac{2 \times 6936}{65.0}} = \sqrt{213.4} = 14.61 \text{ m/s}$$

5.

$$mgy = \frac{1}{2}mv^2$$

$$y = \frac{v^2}{2g} = \frac{5.50^2}{2(9.81)} = \frac{30.25}{19.62} = 1.542 \text{ m}$$

7.

$$W = mgy_{\text{top}} = \frac{1}{2}mv_{\text{takeoff}}^2$$

$$W = 60.0(9.81)0.452 = 266 \text{ J}$$

$$\frac{1}{2}mv_{\text{takeoff}}^2 = 266 \text{ J}$$

$$v_{\text{takeoff}} = \sqrt{\frac{2 \times 266}{60.0}} = \sqrt{8.868} = 2.98 \text{ m/s}$$

9.

$$mgy = \frac{1}{2}mv^2$$

$$mgy = (15.00 + 0.100)(9.81)(0.35) = 51.85 \text{ J}$$

$$\frac{1}{2}mv^2 = 51.85 \text{ J}$$

$$v = \sqrt{\frac{2 \times 51.85}{0.100}} = \sqrt{1036.9} = 32.2 \text{ m/s}$$