

RESPONSE CHARACTERISTICS OF DIFFERENT BUTTERWORTH LOW-PASS DIGITAL FILTERS

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INTRODUCTION

For many biomechanics investigators, the low-pass Butterworth digital filter is an important tool for the processing of biomechanical data. A variety of digital filtering options exist (e.g., zero lag, damping, order, etc.), which influence the filter's ability to respond to changes in the raw data (Winter, 1990). The purpose of this research was to examine the effects of filtering a known set of data with various combinations of these options.

PROCEDURES

An artificial step function was created, using amplitude values of zero and one. A low-pass Butterworth digital filter (Winter, 1990) was then passed over the data according to the specified trial characteristics. The trial characteristics (or filter options) that were compared examined the effect produced by: critically damping and underdamping, zero lag and nonzero lag, varying cutoff frequency and cascading the filter.

To make a zero lag filter one must pass the data through the filter twice — once in the forward direction and once in reverse. Cascading a filter means to run the data through the filter more than once.

Whenever data is run through a filter more than once the effective cutoff frequency of the filter is reduced. Therefore to maintain the correct cutoff when multiple passes of a filter is used you must first adjust the cutoff. The following equations were used to adjust the critically damped and underdamped filters, respectively.

$$f_{crit}^* = \frac{f_{crit}}{\left[\frac{1}{2^{2*n} - 1} \right]^{\frac{1}{2}}}$$

$$f_{under}^* = \frac{f_{under}}{\left[\frac{1}{2^n - 1} \right]^{\frac{1}{4}}}$$

where f_{crit} and f_{under} are the desired cutoff frequencies of the critically damped and underdamped filters,

and f_{crit}^* and f_{under}^* are the adjusted cutoff frequencies

necessary to produced the requested cutoff,

and n is the number of filter passes.

RESULTS AND DISCUSSION

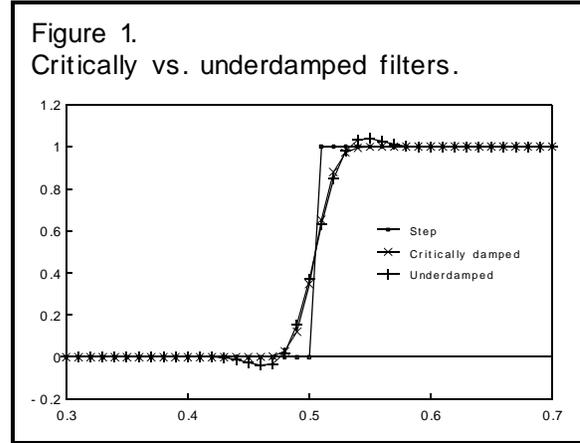


Figure 1 shows the original step function compared to the output from a critically damped and an underdamped filter (both of which are 4th order, zero lag). Sampling rate and cutoff frequency for both filters were set at 100 Hz and 10 Hz, respectively. Notice that, as expected, the critically damped filter does not undershoot or overshoot the data. The underdamped filter does produce an artificial minimum and maximum and takes slightly longer to match up to the original data. It is clear from these results that the critically damped filter is superior to the underdamped filter.

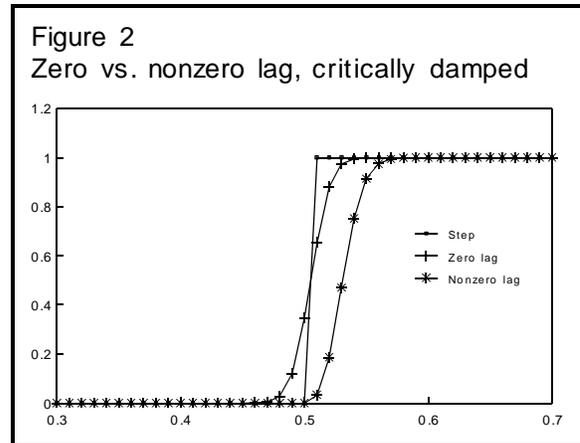
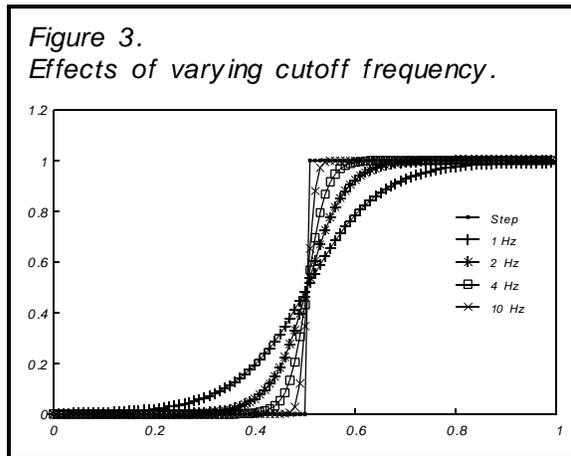


Figure 2 compares the step function to both a zero and a nonzero lag filter. Both filters were critically damped, 4th order and had sampling and cutoff frequencies of 100 Hz and 10 Hz, respectively. A filter which is referred to as zero lag, achieves this by passing over the data twice — once in each direction. This process introduces an equal and opposite phase lag so that the net phase shift will be zero. As is evident here, the nonzero lag filter significantly lags behind the

original data (i.e., is out-of-phase). It should be noted that the zero lag filter begins to respond to the change in the original data before that change actually occurs. It can be said to “anticipate” the data. On the other hand, the nonzero lag filter is not affected until the first change in the original data which inevitably produces a phase lag in comparison to the original signal. Despite the slight anticipation produced by the zero lag filter, its elimination of phase lag makes it clearly superior to the nonzero lag filter for most purposes.

Figure 3 shows the effects produced by changing the cutoff frequency of a zero lag, critically damped filter. Cutoff frequencies of 1, 2, 4, and 10 Hz were applied assuming a sampling rate of 100 Hz. As was expected, varying the cutoff frequency significantly affected the ability of the filter to follow the original step function. It can be seen that the poorest match occurs with the filter which had the 1 Hz



cutoff frequency. With the addition of more high frequency components, the filter’s ability to reproduce accurately the original step function was improved. The cutoff frequency of 1 Hz eliminated almost all of the original harmonics, resulting in a very poor and simplified approximation of the step. Of the four different cutoff frequencies, the best choice for most accurately reproducing the original data was the 10 Hz cutoff. In addition, because of the greater harmonic content, the filter with the highest cutoff frequency is able to get back on track with the original data faster than the filters with lower cutoffs. It should be kept in mind that while producing a better reproduction, a higher cutoff may also be insufficient in eliminating noise from the original data.

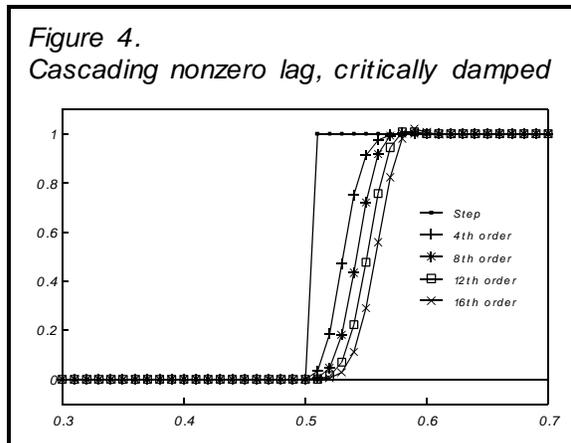
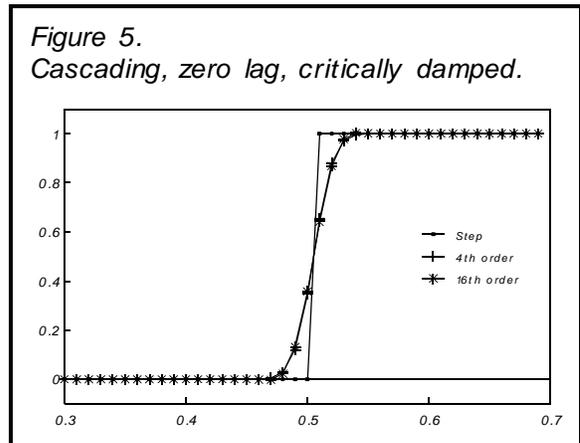


Figure 4 shows the affects of “cascading” a 2nd order, nonzero lag, critically damped filter. This has the effect of increasing the rise time and the order of the filter. After adjusting the cutoff frequency for the number of repetitions, the data were filtered 2, 4, 6 and 8 times with the same filter. This produced a 4th, 8th, 12th and 16th order filter. In all cases the sampling rate and cutoff frequency of 100 Hz and 10 Hz, respectively, were used. It can be seen that the lowest order filter has less phase lag compared to the filters of higher order, with the 16th order filter having the greatest amount of lag.

Figure 5 shows the results of cascading a critically damped, zero lag filter 1 and 4 times, producing 4th and 16th order filters. The equivalent 8th and 12th were not plotted since their results were virtually identical to the others. Note that a zero lag filter, implicitly passes over the data twice — once in each direction. Therefore multiple passes increase the order by four. In Figure 5, it is immediately apparent that there is almost no difference between the



different orders. This result was not expected. What was expected was the pattern shown in Figure 4 for the nonzero lag condition. Increasing the numerical value of the order of the filter should increase its ability to respond to a change in the original data. These results suggest that the order of the filter is irrelevant when it is also zero lag.

CONCLUSIONS

In this investigation, the effect of several different filtering options on the response characteristics of a low-pass Butterworth digital filter were examined. The results have shown the superiority of using a critically damped and zero lag filter. Furthermore, it was apparent that increasing the order of a zero lag filter has little influence on its ability to respond to changes in the original data. Based on the digital filtering options that were examined in this study, it can be concluded that a critically damped, zero lag filter with a cutoff frequency high enough to include the majority of the harmonic components of the wanted signal, should be the combination used to achieve the best possible data smoothing results.

REFERENCES

Winter, DA, *Biomechanics and Motor Control of Human Movement*, Wiley Interscience, 1990.