ON THE USE OF SPLINE FUNCTIONS FOR DATA SMOOTHING*

Abstract - The appropriateness of various numerical procedures for obtaining valid time-derivative data recently reported in the literature (Zernicke et al., 1976; McLaughlin et al., 1977; Pezzack et al., 1977) is discussed. A case for the use of quintic natural splines is presented, based on the smoothness of higher derivatives and flexibility in application.

Fig. 1. Second derivatives of cubic and quintic spline approximations to vertical jump data from Miller and Nelson (1973). CUBIC = cubic spline; QUINT = quintic spline; VALID = ground reaction force values.

INTRODUCTION

In recent years several papers have been published which deal with the computation of valid derivative measures from digitized position–time data. The use of cubic splines was advocated by Zernicke et al. (1976) and this method has been shown to be superior to traditional polynomial and finite difference techniques (McLaughlin et al., 1977). Pezzack et al. (1977) have shown that valid derivative data can also be obtained by digital filtering followed by finite differences differentiation. These authors further suggest that the placement of “knots” in the use of cubic splines necessarily encumbers that procedure and that a spline is likely to oversmooth in regions of complex change. However, the spline approximation procedure commonly employed is that of Reinsch (1967, 1971) which provides a natural spline function that is minimized under the boundary condition that the fit is within the “accuracy of measurement”

\[ Q = \sum_{i=1}^{n} \left[ g(t_i) - y_i \right]^2 \leq S, \]

where the \( \delta y_i \) are standard errors of measurement and \( S \) is a parameter that controls the extent of smoothing. Such a spline function has the properties that

\[ g'(t_1) = g'(t_N) = 0; \]

it has \((2m-2)\) continuous derivatives and is the smoothest possible function that fits the data within the specified accuracy (Wold, 1974).

CUBIC vs. QUINTIC SPLINES

While cubic splines \((m = 2)\) are conventionally used for data smoothing purposes, there are some inherent weaknesses in modelling biomechanical data with a spline of this order. First, the third derivative (jerk) has jump discontinuities, and to assume that forces acting within the body act in a non-smooth manner would seem to be inappropriate. Secondly, the boundary conditions in (3) often impose unrealistic end-values for the second derivative \( g'' \). To illustrate these points, Figs. 1 and 2 present time-derivative data plots for cubic \((m = 2)\) and quintic \((m = 3)\) spline approximations to the vertical displacement of the mass center of a person during a standing jump take-off. The position–time data were taken from Table 4.3 in Miller and Nelson (1973). A digitized representation of the ground reaction force (normalized) has been superimposed on the two acceleration curves (Fig. 1).

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† Spline functions are piecewise polynomials of some degree \( n \) joined at points called knots in such a manner as to have \( n-1 \) continuous derivatives. A spline of odd degree \((n = 2m-1, m = 1, 2, \ldots, \text{etc.})\) is called a natural spline if it is given in each of the two intervals \((-\infty, t_1], (t_N, \infty)\) by some polynomial of degree \( m-1 \) (rather than \( 2m-1 \)) or less (Greville, 1969).
and highlights the more appropriate fit of the quintic model in terms of the smoothness of the function and valid terminal values. The discontinuities inherent in the third derivative of the cubic spline function lends further support to this contention (Fig. 2).

The validity of quintic spline approximations is further demonstrated by approximation to data extracted from Pezzack et al. (1977). As can be seen in Fig. 3, the quintic spline provides an excellent fit to the raw angular displacement data extracted from film by these authors, and the second derivative conforms closely to a digitized representation of their analogue acceleration trace (Fig. 4).

DISCUSSION

It is acknowledged that Pezzack et al. (1977) also obtained excellent conformity between digital and analogue acceleration-time curves, but the usual requirement of equidistant abscissae for digital filtering and finite difference calculus algorithms can be an undesirable limitation, given that some data acquisition systems do not provide exact time increments. Further, the provision for time-dependent error variance estimates in the least squares splining procedure avoids the assumption implicit in digital filtering that the "noise" is uncorrelated with the "signal". Spline functions also have the desirable property (cf. polynomials and recursive filters) that their behaviour in one region may be totally unrelated to their behaviour in another region (Rice, 1969).

Finally, some useful generalizations of spline functions have been formulated in recent years, based on variations to the boundary conditions stated in equation (3). Of these the periodic spline \( g(j) = g_{2m}(j) \) \( (j = 0, \ldots, 2m - 2) \), the parametric spline and the cyclic spline (smooth curves through a set of arbitrary points \( (x_i, y_i), i = 1, 2, \ldots, N \) have obvious application for kinematic analyses (see Späth, 1974).
Fig. 4. Second derivative of quintic spline approximation to film data from Pezzack et al. (1977). QUINT = second derivative of quintic spline function; VALID = accelerometer data.

References


