ON PRACTICAL EVALUATION OF DIFFERENTIATION TECHNIQUES FOR HUMAN GAIT ANALYSIS

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Abstract—Numerical differentiation of noisy measurement data represents a problem frequently encountered in the field of gait analysis. There are two major determinants of the quality in calculated derivatives, namely the quality of the measurement data and the quality of the used differentiation technique.

The quality of the measurement data, with respect to the maximum precision that can be obtained in calculated derivatives, is discussed with the help of an error formula valid for all differentiating techniques. It is verified that high precision can be obtained in the calculated second derivatives even with crude techniques, provided that the quality of the measurement data are good enough. This point is illustrated by the differentiation of film data from Pezzack et al. (1977), using a least squares polynomial fitting.

For the evaluation and comparison of different techniques for numerical differentiation it is recommended that measurement data with a considerable amount of noise is used, and that the quality of calculated derivatives are evaluated not only by visual inspection of graphical displays, but also with the use of a quantitative criteria, such as the root mean squares error.

INTRODUCTION

In recent years several papers have been published about various techniques for numerical differentiation of data from human gait measurements. The topic of this paper is practical evaluation of such techniques. The quality of the measurement data, which is a major determinant of the achievable precision in calculated derivatives, is discussed from a general point of view. As an example it is demonstrated that the data presented in Pezzack et al. (1977) are not suited for direct comparison between differentiating techniques since the quality of these data is "too high" for the purpose.

It is important to bear in mind that there are two factors that affect the quality of calculated derivatives:

1. The quality of the measurement data.
2. The quality of the differentiation technique.

Although the first of these factors determines the maximal accuracy that can be obtained in calculated derivatives, it has been little discussed in the literature. Actually, if there is too much noise superimposed on the measurement data and/or an inadequately low sampling rate, then there is no differentiation technique that can produce good estimates of the derivatives. If, on the other hand, the noise level is low and the sampling is high enough, then almost any differentiation technique will do the job.

This can be understood if the differentiation filters are regarded as noise amplifiers. The noise present in the measurement data is amplified in the filter but in most cases there is no substantial noise generated in the process. One useful measure of the quality of a filter is the noise transmission which can be defined as the amplification of the noise standard deviation in the filter when the input noise is white. For linear filters, to which this paper will be restricted, this measure is independent of the input noise level. Therefore, the noise level in the computed derivatives will be relatively low, even if the filter has a high noise transmission, provided that the measurement noise level is sufficiently low.

As an example of this the film data presented in Pezzack et al. (1977) will be studied. These data have been used for the evaluation of differentiation techniques in several recent papers. In Pezzack et al. (1977), Gustafsson and Lanshammar (1977), Soudan and Dierckx (1979), and Wood and Jennings (1979), different techniques for numerical differentiation were proposed. Pezzack et al. (1977) advocated low pass filtering (2nd order Butterworth filter) prior to differentiation by finite difference approximation, while Gustafsson and Lanshammar (1977) proposed a technique based on least squares error minimization and involving a Taylor series approximation of the signal. Soudan and Dierckx (1979) used cubic splines approximation and in Wood and Jennings (1979) a quintic splines approximation was used.

In all these papers the results from double differentiation of the mentioned film data were presented. The results from the differentiation were compared to accelerometer data also presented in Pezzack et al. (1977), and the fit was excellent in all cases.

The conclusion to be drawn is not that all the proposed methods are equally well suited for numerical differentiation of noisy data. Instead, these results are effects of the very good quality of the used film data. The quality of these data, in terms of noise level, are in fact so good that they can be differentiated twice even

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with crude techniques, provided that the filter parameters are properly adjusted to the characteristics of the data.

This point will be illustrated with the use of least squares polynomial fitting. It will be verified that good estimates of the second derivatives can be obtained from the film data using this relatively simple method. The quality of the film data will also be further analyzed with a formula presented in Lanshammar (1980a). This formula gives the maximum precision that can be obtained in estimated derivatives when the characteristics of the measurement data are known. From this analysis it is clear that theoretically the film data are very well suited for double differentiation with good precision in the result.

To make the film data more similar to data obtained from human gait measurements, and to provide a tougher test for differentiation techniques, the film data from Pezzack et al. (1977) can be contaminated by the addition of more noise. This has been done and results are presented from double differentiation of this data by least squares polynomial fitting. The contaminated data are also presented. It is suggested that other differentiation techniques should be tested on this data set as well.

For the evaluation of and comparison between differentiation techniques, the qualitative evaluation obtained from visual inspection of calculated and measured accelerations must be complemented with a quantitative criterion. Such a criterion should be based on a measure of the closeness of fit. Although several such measures could be chosen, the root mean square error is proposed since it is probably the most commonly accepted. Thus the suggested criteria function is

\[ V_{\text{RMS}} = \frac{1}{N} \sum_{i=1}^{N} (y_i^k - \hat{y}_i)^2 \]

where \( y_i^k \) is the value of the \( k \)th order derivative at sample number \( i \), as obtained by numerical differentiation. \( \hat{y}_i \) are independent reference data, as e.g. the accelerometer data in Pezzack et al. (1977). Finally \( N \) is the number of samples available for the test. When several differentiation techniques are compared the "best" technique with respect to this criterion is the one for which \( V_{\text{RMS}} \) assumes the smallest value.

The use of a criterion is especially important when the quality of the data is high, which is true for the data in Pezzack et al. (1977).

Another quantitative entity that can be used as a quality index for differentiating filters is the Relative Noise Amplification (RNA). It is a dimensionless entity describing the noise reduction effectivity of filters. This concept is introduced in Lanshammar (1980b) and will not be further discussed in the present paper.

LEAST SQUARES POLYNOMIAL FITTING

In Fig. 1(a) the film data from Pezzack et al. (1977) are displayed. Figure 1(b) illustrates the corresponding accelerometer data. In Fig. 2(a) the result from a second order least squares polynomial fitting and double differentiation of the polynomial is compared to the accelerometer data. The polynomial fitting was done for five successive data points and the second derivative of the polynomial at the midpoint was used as an estimate of the acceleration at that point. In this way polynomials were fitted around each data point except for the first two and the last two points for which no estimates were obtained.

The characteristics of differentiating filters obtained by local least squares polynomial fitting have been analyzed (Lanshammar, 1980b). Among other things the bandwidth and the noise transmission of the resulting filters were computed. This analysis showed that the filter corresponding to a second order polynomial fitted to five data points has the approximate bandwidth 0.15/\( T \) Hz where \( T \) is the sampling interval.

For the film data, the sampling interval was 0.0201 s which means that the filter bandwidth was 7.5 Hz. The noise transmission for the same filter was 1.3 x 10^{-3} s^{-2}.

In Fig. 2(b) the degree of the polynomial has been increased. In this case fourth order polynomials were fitted to nine data points. For this filter the bandwidth was 8.5 Hz and the noise transmission was 1.1 x 10^{-3} s^{-2}. By using a higher order polynomial a sharper cut off and better damping outside the passband was
obtained. Thereby a slightly lower noise transmission simultaneously with a higher bandwidth could be obtained, as compared to the second order polynomial fitted to five data points.

The fitting displayed in Fig. 2(b) is fully comparable to the results obtained in the four references cited in the introduction. Even better results were obtained by more careful choice of polynomial for the differentiation. In Fig. 2(c) 8th order polynomials were used and in this case fitted to 17 data points.

The bandwidth of this filter was 9.0 Hz and the noise transmission was $0.94 \times 10^3 \text{s}^{-2}$. In this case we have obtained a further reduction in the noise transmission and an increased bandwidth. The drawback with a higher order filter as compared to a low order filter is that the computation time increases linearly with the order of the filter.

In Fig. 3 the properties of the discussed filters are further illustrated by graphs of the filters frequency response, $H(\omega)$. Normalized amplitude curves, $|H(\omega)|T^2$, for the twice differentiating filters are plotted versus normalized frequencies, $\omega T$. Note that the scaling on the vertical axes are slightly different in the three curves.

As can be seen in Fig. 3 the amplitude curves have a sharper cut off outside the pass-band for higher order filters (Figs. 3b and 3c). Also the sidelobes, characteristic for polynomial filters, are smaller for the higher order filters. This explains why the noise transmissions have smaller values for higher order polynomials than for polynomials of low order.

### QUALITY OF THE MEASUREMENT DATA

The noise level in the film data was estimated by the residuals in least squares polynomial fittings. The good correspondence between the accelerometer data and the differentiated polynomials exhibited in Fig. 2(b) means that it is essentially the signal that has been fitted while the noise has been left untouched. Therefore it is not unreasonable to estimate the measurement noise variance with that of the residuals. For 134 samples, the variance of the residuals was $1.7 \times 10^{-6} \text{rad}^2$, corresponding to the standard deviation $0.0013 \text{rad}$. (In this case the fourth order polynomial fitted to 9 data points was used, but similar results were obtained for 6th and 8th order polynomials.) This is indeed a very good precision for angular data. This figure can be compared to results from Soudan and Dierckx (1979), concerning the same data. Here a smoothing factor $S = 0.0003$ was used, which for the 142 measurement samples corresponds to the measurement noise variance $0.0003/142 = 2 \times 10^{-6} \text{rad}^2$. This is in good accordance with our result.

For comparison we will compute a typical variance of the error in the angular data, when the angles are calculated from displacement measurements during human gait. Suppose that two landmarks are fixed at a body segment. The distance between the landmarks is 1 m. Assume further that the variance of the coordinate error in any fixed direction is $\sigma_y^2$, and that the errors at the two landmarks are independent and have mean value zero. Further, planar analysis is assumed. When the coordinate error is small as compared to l, it can easily be verified that the variance of the angular error

$$
\sigma_\theta^2 = \frac{2\sigma_y^2}{l^2}.
$$

![Fig. 2. The angular data twice differentiated by least squares polynomial approximation (solid line with crosses). Accelerometer data superimposed for reference (solid line). (a) 2nd order 5 points approximation. Filter bandwidth 7.5 Hz. (b) 4th order 9 points approximation. Filter bandwidth 8.3 Hz. (c) 8th order 17 points approximation. Filter bandwidth 9.0 Hz.](image-url)
Fig. 3. Frequency response for double differentiating filters obtained by least squares polynomial approximation. Amplitudes and frequencies are normalized with respect to the sampling interval, T. The bandwidths of the filters are also marked. (a) 2nd order 5 points approximation. (b) 4th order 9 points approximation. (c) 8th order 17 points approximation.

Now, the distance between landmarks on a human body is probably most often below 0.5 m. When the measurement includes both legs and trunk, the measurement field is at least approximately 2 m. If the standard deviation of the error is 0.1%, which corresponds to high quality data, we get

$$\sigma_2 = 0.002 \text{ m}$$

and thus

$$\sigma^2 \geq \frac{2(0.002)^2}{(0.5)^2} = 3.2 \times 10^{-5} \text{ rad}^2 = \sigma_2 \geq 0.006 \text{ rad.}$$

Note that for shorter distances between the landmarks or more noise on the displacement data, the angular error will increase.

The standard deviation calculated above is thus more than four times the error found in the data from Pezzack et al. (1977), which was 0.0013 rad. Therefore it is not ascertained that a differentiation technique that behaves well for Pezzack's data, will be suitable for the differentiation of gait data.

To obtain the precision depicted in the data from Pezzack et al. (1977), the displacement data must have the precision

$$\sigma^2 = \sigma_2^2 \frac{\sigma_2^2}{2} \leq \frac{(0.0013)^2(0.5)^2}{2} = 2.1 \times 10^{-7} \text{ m}^2$$

or

$$\sigma_2 \leq 0.46 \times 10^{-3} \text{ m.}$$

To the best of the author's knowledge this precision has not yet been obtained during any human gait measurements.

**QUALITY OF THE DIFFERENTIATED DATA**

From the formula presented in Lanshammar (1980a), it is possible to calculate the maximal precision that can be obtained in the calculated second derivatives. The formula applies when the measurement noise is white. According to this formula the minimal variance in the second derivatives ($\sigma^2$) is

$$\sigma^2 \leq \frac{\sigma_2^2 T(2\pi v_s)^4}{5\pi}.$$  (1)

Here, $\sigma_2^2$ is the variance of the measurement noise, $T$ (s) is the sampling interval, and $v_s$ (Hz) is the signal bandlimit. It is assumed that the signal is strictly bandlimited so that no signal is present above the bandlimit $v_s$ Hz.

The formula applies for all differentiating techniques that give unbiased estimates of the second order derivatives.

In the present case the sampling interval was 0.0201 s, and we get

$$\sigma_2^2 \geq \frac{1.7 \times 10^{-6} \times 0.0201(2\pi v_s)^4}{5\pi} = 2.13 \times 10^{-8} \text{ rad}^2/s^2.$$  

Strictly speaking the bandlimit $v_s$ does not exist, but according to the results in Fig. 2(c) where the filter bandwidth was 9.0 Hz there is certainly not much signal power above that frequency.
The minimal variance in the second order derivatives was calculated for some different bandlimits. The results are presented in Table 1.

Thus, for signal bandlimits between 7 and 10 Hz the standard deviation in calculated second derivatives can theoretically approach \( \sim 1 \text{ rad/s}^2 \) provided a good differentiating filter is used. This corresponds to 1% of the maximal angular acceleration, which according to the accelerometer data is about 100 rad/s\(^2\).

The conclusion from this analysis is that the film data from Pezzack et al. (1977) has a very high quality with respect to the possibility of calculating second derivatives with good precision.

The lower limit on the variance above is obtained for a differentiating filter that acts as a perfect differentiator below the signal bandlimit and that has zero output for frequencies above the bandlimit. For practical filters, the noise level is larger if their performance is close to an ideal differentiator below the bandlimit. The reason for this is that the cut off cannot be made infinitely sharp and therefore a small part of the noise power outside the passband will always be left in the calculated derivatives.

For the filter in Fig. 2(c) the noise transmission was 0.94 \( \times 10^3 \) per s\(^2\). In this case the variance of the measurement noise was 1.7 \( \times 10^{-6} \) rad\(^2\), which gives a noise variance in the second derivative

\[
\sigma_2^2 = 1.7 \times 10^{-6}(0.94 \times 10^3)^2 = 1.50 \text{ rad}^2/\text{s}^4
\]

\[
(\sigma_2 = 1.23 \text{ rad/s}^2).
\]

Since the bandwidth of this filter was 9.0 Hz, the noise level in this case is slightly above the minimal value (1.12 rad/s\(^2\)).

The practically obtained noise variance in the estimated second derivative was also calculated. This variance was estimated by

\[
\hat{\sigma}_2^2 = \frac{1}{125} \sum_{i=1}^{134} (\hat{\phi}_i - \hat{\phi}_i)^2 = 11.41 \text{ rad}^2/\text{s}^4 (\hat{\sigma}_2 = 3.38 \text{ rad/s}^2)
\]

where \( \hat{\phi}_i \) is the acceleration value at sample number \( i \) according to the accelerometer data, and \( \hat{\phi}_i \) is the double differentiated film data. Exact numerical values for the accelerometer data were not available. Instead they were measured from a graphical display of the data obtained by D. Winter. The summation was done over the samples 9–134, since no estimates of the acceleration were obtained for the first 8 and the last 8 of the 142 available samples.

The practically found noise variance in the second derivative (11.4 rad\(^2\)/s\(^4\)) was larger than that foreseen by the noise transmission of the filter. However, when the variance was calculated over the first 60 usable samples the result was

\[
\hat{\sigma}_2^2 = \frac{1}{60} \sum_{i=1}^{60} (\hat{\phi}_i - \hat{\phi}_i)^2 = 3.69 \text{ rad}^2/\text{s}^4 (\hat{\sigma}_2 = 1.92 \text{ rad/s}^2)
\]

This explanation for this is that the angle was varying much slower in the first half of the measurement than in the second half (see Fig. 1). During the second half of the measurement the signal contained more high frequency components which caused systematic errors when low pass differentiated. The practically calculated variance is a measure of the total error, including the systematic error in the differentiation process. Therefore, it is not surprising that the theoretical variance, calculated from the filters white noise transmission is smaller.

Even when only the first 60 samples are taken into account there is a significant difference between the practical and the theoretical variance. The above reasoning about the systematic error due to the filter applies also in this case, although it is less important during the first half of the measurement. Another source of error is that the noise transmission gives the standard deviation of the output noise when the input noise is white, i.e. uncorrelated. In practice the noise is not exactly white, and with colored noise the output noise level may increase.

### Table 1. Minimal values of the noise variance and standard deviation in calculated second order derivatives for the data from Pezzack et al. (1977) and for different assumed signal bandlimits (\( v_0 \))

<table>
<thead>
<tr>
<th>( v_0 (\text{Hz}) )</th>
<th>( \sigma_2^2 (\text{rad}^2/\text{s}^4) )</th>
<th>( \sigma_2 (\text{rad/s}^2) )</th>
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**DIFFERENTIATION OF FILM DATA WITH ADDED NOISE**

To make the film data more realistic and suitable for evaluation of different differentiation techniques, pseudo-random white noise was added to the raw film data. The resulting data are presented in Table 2, for others to test their differentiation techniques. The added noise has zero mean value and the standard deviation is 0.006 rad. The standard deviation of the total noise was therefore \( \sqrt{0.006^2 + 0.001^2} = 0.006 \) rad. In this way the resulting data has a precision comparable to what can be obtained from human gait measurements.

Now on a graphical display the new data looks just as smooth as the original data in Fig. 1. However, the added noise has a profound effect on the calculated second derivatives.

In Fig. 4(a) the result from double differentiation of the data with the 8th order of 17 points polynomial filter is presented. In this case the fit is not at all comparable to the fit in Fig. 2(c), where the same filter was used on less noisy data. To understand this had result it is interesting to calculate the minimal error that can be obtained in second derivatives according to the error
Table 2. Angular data from Pezzack et al. (1977) with added white noise (standard deviation 0.006 rad). Sampling interval 0.0201 s

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Fig. 4. The angular data with added white noise (standard deviation 0.006 rad) and twice differentiated by least squares polynomial approximation (solid line with crosses). Accelerometer data superimposed for reference (solid line). (a) 8th order 17 points approximation. Bandwidth 9.0 Hz. (b) 8th order 29 points approximation. Bandwidth 4.9 Hz.

According to the minimum error formula the minimal error due to measurement noise in second derivatives is 1.15 rad/s² when the signal bandwidth is 4.9 Hz. This minimal error provides a reference value for the evaluation of the practically obtained error, 4.29 rad/s².

Thus, to obtain an acceptable result in the slowly varying part of the measurement, it was necessary to decrease the bandwidth of the filter. Doing this, one has to accept increasing errors in the faster varying, latter part of the measurement if a time invariant filter is to be used. However, the overall fit was improved in Fig. 4(b) as compared to Fig. 4(a).

CONCLUSION

The difference in behaviour between various techniques for numerical differentiation is of practical importance only if the quality of the measurement data is relatively poor. To analyze the quality of measurement data, as far as achievable precision in the calculated derivatives is concerned, a formula presented in Lanshammar (1980a) can be used.

The precision of the film data presented in Pezzack et al. (1977), for the testing of numerical differentiation techniques, is very high. Due to this several techniques can reproduce the simultaneously measured accelerometer data excellently. This is a problem because in this case it is not possible to compare the different techniques by visual inspection of the calculated acceleration graphs.

Also, it has been argued in this paper that displace-
ment data obtained from human gait measurements will generally be contaminated with noise whose standard deviation is more than four times that of the data from Pezzack et al. (1977). Therefore, it is not guaranteed that a differentiation method that gives excellent results for Pezzack's data will be suitable for the differentiation of data from human gait measurements.

Still, it is not recommended to abandon the data from Pezzack et al. (1977) because the existence of simultaneous accelerometer data makes them very valuable. They represent high quality independent reference data that are difficult to obtain from human gait experiments. Instead the following actions are proposed.

(1) For practical evaluation of the quality of techniques for differentiation, when reference data for the derivatives are available, a quantitative measure of the goodness of fit should be used. The root mean square error is proposed as such a measure.

(2) As a complement to the data presented in Pezzack et al. (1977), it is proposed that the data in Table 2 are used as test data for differentiation techniques applicable to human gait analysis. The data in Table 2 are Pezzack's with added pseudo-random white noise (Gaussian distribution, standard deviation 0.006 rad). The accelerometer data from Pezzack et al. (1977) still applies as reference data.

When 8th order, 29 points polynomials were fitted to the data in Table 2 and used for the estimation of the second order derivative, the resulting root mean square error was 4.29 rad/s².

Finally, it must be pointed out that this paper has dealt only with one quality aspect of techniques for numerical differentiation, namely the precision in the calculated derivatives. To compare different techniques one must also consider other practical questions. Some important characteristics are:

(i) Length of the computer program.
(ii) Computer time needed for the differentiation.
(iii) Difficulty to tune the differentiating filter, i.e. to determine suitable filter parameter for a specific set of measurement data.

Acknowledgement—I wish to thank D. Winter, University of Waterloo, Waterloo, Ontario, for supplying me with displacement and accelerometer data.

REFERENCES


