Crash Tests and the Head Injury Criterion

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Abstract

The Head Injury Criterion (HIC) model has been developed to measure quantitatively the head injury risk in crash situations. Using a computer algebra system (here MAPLE), students are able to analyse results reported from a real crash test carried out with Mercedes-Benz cars. The discussion of the model provides first an excellent motivation for the Riemann integral, serves secondly as a piece of real consumer enlightenment, and demonstrates thirdly that the need to (over)simplify reality may produce hardly meaningful results, despite considerable mathematical expenditure.

The negative acceleration during a crash

Fortunately, today many consumers pay more attention to the safety of their cars than to fancy spoilers and aluminium rims. Crash tests (Fig. 1) can give valuable hints on advantages or disadvantages of construction.

A number, denoted \( HIC \), is often quoted in tests and reports without it being clear what that number is. To clarify this idea is, on the one hand, a case of real consumer enlightenment, and on the other an excellent motivation for the Riemann integral.

The \( HIC \) (Head Injury Criterion) is intended to judge the head injury risk quantitatively. In the case of an accident, the head load results from too high (negative) acceleration (or better, deceleration) values during the crash. Construction features such as crumple zones and airbags are employed to prolong the period of braking on the driver's body and thus to lower the deceleration during the crash below critical values. The dummies used in crash tests have several sensors fixed to the head area which record the absolute value of the deceleration and its dependence on time. It is plausible that this head load is higher the larger the value of the deceleration and the longer the deceleration lasts. Some empirical values are known from tests with animals (not ethically without question), with corpses, and—especially remarkable—from the evaluation of injuries to boxers. The goal is to get a quantitative evaluation of the head load from the recorded \( a-t \) curve.

Velocity and deceleration

If you show students a qualitative \( a-t \) diagram like that in Fig. 2, they will usually propose as a first model to measure the area below the curve (they even do this before the introduction of the integral). However, the deceleration curve of Fig. 3, which contains the same area, but obviously looks "more dangerous", shows that this first proposal is not useful. It should be discussed with the students what the deceleration curve measured during the crash actually describes. If a car changes its velocity during the time \( \Delta t \) from \( v_1 \) to \( v_2 \), the quotient

\[
\bar{a} = \frac{v_2 - v_1}{\Delta t}
\]

is the average acceleration (<0 if the car slows down, >0 if it speeds up). As usual, the quotient of differences leads to the derivative in the limit \( \Delta t \to 0 \), going from the average rate of change to the local rate of change; here the instantaneous acceleration \( a(t) = v'(t) \).

The value \( a(t) \), measured in \( \text{m/s}^2 \), shows in an intuitive way how much (more or less) the velocity, measured in \( \text{m/s} \), will increase or decrease during the next second. Conversely, given the acceleration curve (as from measurement in the crash test) the \( v-t \) curve can be reconstructed from the rates of change. Accelerating with velocity \( v_0 \) starting at time \( t_0 \), we have \( t > t_0 \)

\[
v(t) = v_0 + \int_{t_0}^{t} a(\tau) \, d\tau.
\]

Deceleration during the crash

With "normal" braking it is quite acceptable to calculate using a constant deceleration (cf. [1; p. 28], [2]). Modern cars reach values from \( 8 \text{ m/s}^2 \) to \( 11 \text{ m/s}^2 \), that is, about the terrestrial acceleration due to gravity, \( g \). Racing cars reach values up to \( 5g \) (which is possible using special mixtures of
the tyre’s rubber and the high values of the downward pressure caused by the aerodynamic shaping of racing cars. For crash tests, one starts with velocities of 30 mph (≈ 48.3 km/h ≈ 13.4 m/s). In “normal” braking, the process lasts about 1.5–2 s with constant deceleration. Let your students absorb that fact for future reference! During a crash, the braking process lasts about 100–200 ms, and the life-threatening deceleration peaks have a duration of about 10 ms. The process of deceleration of the head area needs to be changed from the dangerous form of Fig. 3 to the regular form of Fig. 2 with \( a \) values clearly less than 100g. Crumple zones and other constructive devices are used to achieve this. To evaluate the complete process of deceleration one tries to weight the decelerations appropriately. Figures 4 and 5 show the result of a crash test. Both are of the same car.
type, the former E-class of Mercedes-Benz (model W 123). Figure 4 shows the crash performed without an airbag, Fig. 5 with an airbag. The meanings of the marked rectangles and the values of $HIC$ and $A-3ms$ will be explained in the following. Ask your students to describe the two curves.

Severity Index and the Head Injury Criterion
The first model used in practice was the Severity Index $SI$. The value of the deceleration was weighted by a power $n$, the value of which depended on the part of the body according to empirical experience. For the head, $n = 2.5$ was chosen. The $SI$ was defined in mathematical form by

$$SI = \int_0^T a(t)^n \, dt,$$

with $n = 2.5$ for the head, and with $T$ equal to the complete duration of the deceleration having an effect on the head (cf. [3] for the definition of the $SI$ and for the following definition of the $HIC$). This formula is already understandable before the introduction of the integral and can be used to lead to the integral.

The validation of the model was not satisfactory for comparison of different car types and different accident situations. Adaptation of the empirical data led to the currently used $HIC$ value as a revised model. First, we look at the mean acceleration between two times $t_1$ and $t_2$ (this simul-
Fig. 6

\[ a = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} a(t) \, dt. \]

The factor \((t_2 - t_1) \cdot a^{2.5}\) takes into account both duration and weighted value of the deceleration for the time interval \(t_1\) to \(t_2\). The maximum of all such numerical values is the Head Injury Criterion

\[ HIC = \max_{t_1, t_2} \left( (t_2 - t_1) \left( \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} a(t) \, dt \right)^{2.5} \right). \]

Apart from the condition \(0 < t_1 < t_2 < T\), the starting point \(t_1\) and the time period \(\Delta t = t_2 - t_1\) are arbitrary at first. To be able to evaluate the measured curve practically, one makes some simplifying model assumptions:

- We demand \(t_2 - t_1 \leq 36\) ms. Longer deceleration times do not increase the injury risk, according to experience.
- Peak acceleration values must last 3 ms. This requirement has reasons of measurement technique and is supported by the assumption that decelerations of shorter duration do not have any effect on the brain.

The computation of the HIC from a measured curve, which actually consists of a huge number of finally interpolated points, is still very laborious and is not possible without high-speed computers. The calculation is not done using calculus methods (one does not have an equation of the curve; later we will do it nevertheless) but with numerical methods. In both Figs 4 and 5, the marked rectangle corresponds to the \(HIC\)-determining maximum-interval (cf. Fig. 6). The value A-3ms describes the maximal deceleration value lasting at least 3 ms.

From the two measurements, we calculate as follows.

- Fig. 4: \(HIC = 681.85\); the rectangle corresponding to Fig. 6 has the basis \(t_1 = 63\) ms, \(t_2 = 99\) ms, \(\Delta t = 36\) ms and \(\ddot{a} = 89.65g\).
- Fig. 5: \(HIC = 307.84\); the rectangle corresponding to Fig. 6 has the basis \(t_1 = 58\) ms, \(t_2 = 94\) ms, \(\Delta t = 36\) ms and \(\ddot{a} = 43.47g\).

The airbag decreases the HIC from 682 to 308. Incidentally, what is the unit of the HIC? It is used as an unitless number in the publications of the motor press.) The A-3ms value decreases from approximately 90g to approximately 43g. In both cases, the maximum appears for \(\Delta t = 36\) ms. The same holds true for all other crash tests of which I have seen the measurements up to now.

Model calculations for the HIC

The aim of the following is a better understanding of the complicated formula for the HIC. We take the deceleration as a non-negative real function \(a : \mathbb{R} \rightarrow \mathbb{R}, t \rightarrow a(t)\), which should of course have all "desirable" characteristics such as continuity and differentiability. To estimate the HIC, first we calculate the average deceleration \(\ddot{a}\) for the time interval from \(t\) to \(t + d\) for arbitrary values \(t \in \mathbb{R}\) and \(d \in \mathbb{R}\). Then we weight it with the power 2.5 and multiply it with the effective time \(d\) to \(da^{2.5}\). The maximum of all such values is the HIC. We simulate this analytically with a function \(H\):

\[ H(t, d) = d \left( \frac{1}{d} \int_{t}^{t+d} a(\tau) \, d\tau \right)^{2.5}. \]

This formula gives the unit of the HIC if the time \(t\) is measured in seconds and the deceleration \(a\) in g, then the unit of the HIC is \(sg^{2.5}\) (which is usually omitted).

As is common in school, we can understand \(H\) as a family of functions with variable \(t\) and parameter \(d\). Of course, it is more useful to imagine \(H(t, d)\) as a surface of which the highest point above the \(t-d\) plane is the HIC. To establish the existence of that point, one considers (as a good repetition of the basic ideas) the following:

\[ H(t, d) > 0 \text{ for any } t \in \mathbb{R} \text{ and } d > 0. \]

For fixed \(t\) one has \(H(t, d) \rightarrow 0\) for \(d \rightarrow 0\) and for \(d \rightarrow \infty\).

For fixed \(d\) one has \(H(t, d) \rightarrow 0\) for \(t \rightarrow -\infty\) and for \(t \rightarrow \infty\).

A possible way to calculate the HIC is estimation of the global maximum \(H(d)\) of \(H\) (for fixed \(d\)) and then estimation of the maximal \(H(d)\) for \(d > 0\). Of course, that is only possible for very simple functional forms of \(H\). A useful treatment
for any \( H \)-function is the following graphical method in which the estimation of the maximum of the three-dimensional \( t\cdot d\cdot H(t, d) \) surface reduces to a two-dimensional problem. One draws families of functions \( H(t, d) \) with variable \( t \) and parameter \( d \) (or with variable \( d \) and parameter \( t \)) and with their help estimates with given accuracy the desired maximum \( HIC \). In any case, the use of a computer algebra system is necessary. We have been working with MAPLE.

### Model functions for the Mercedes-Benz crash tests

The essential process of deceleration in Figs 4 and 5 lies between 0 and 160 ms. A curve like the one in Fig. 5 can be represented by a rational function \( a \) defined as

\[
a(t) = \frac{b}{(t - c)^2 + d}.
\]

One needs the adequate addition of two of those terms for the curve in Fig. 4. To adapt the respective coefficients, we magnified Figs 4 and 5 and made tables of the values at intervals of 5 or 2.5 ms (cf. Table I; \( a_w \) means with airbag, \( a_o \) means without airbag). Using these values, we plotted the measured curves as polygons with the help of MAPLE.

The \( a \) functions have been fitted to the polygons through suitable modification of the coefficients. Figure 7 shows the result for the crash with airbag, Fig. 9 for the crash without airbag; the measured points are connected with thin polygons, the model curves \( a_w \) and \( a_o \) are drawn with heavy lines.

We choose \( a_w \) as a fitting function for the crash with airbag:

\[
a_w(t) = \frac{22000}{(t - 74)^2 + 500} \quad (t \text{ measured in ms, } a \text{ in g; cf. Fig. 7}).
\]

For the crash without airbag, we first modelled the two parts of the curve with functions \( f \) and \( g \) of our type. This is given in Fig 8. We took

\[
f(t) = \frac{21500}{(t - 70)^2 + 500}, \quad g(t) = \frac{1800}{(t - 92.5)^2 + 18}.
\]
Superposition and repeated fitting lead to the model function $a_0$ drawn in Fig. 9, with

$$a_0(t) = \frac{16400}{(t - 68)^2 + 400} + \frac{1480}{(t - 93)^2 + 18}.$$

Graphical estimation of the HIC for the model functions
The manipulation of such terms is relatively laborious, so that one has to use a computer algebra system (CAS) like MAPLE or DERIVE. With the simpler term $a_m$, it is possible to discuss the curve with the help of the derivatives and so to get the HIC. A CAS such as MAPLE is powerful enough, though one should compare the required power and the gain!

In any case, it is more useful to inspect the functions $H_w(t, d)$ and $H_0(t, d)$ with graphical methods and thus to calculate the HIC for our model functions $a_w$ and $a_0$. For this, we use the ability of MAPLE (or any other CAS) to draw 2-D and 3-D graphs. The graphs of the HIC functions $h_w$ and $h_0$ can be understood as surfaces above the $t$-$d$ plane. Figures 10 and 11 represent those surfaces.

One recognises clearly the distinct maxima of the surfaces. To be able to calculate them more exactly, MAPLE has additionally drawn the respective two-dimensional parameter curves of $H_w(t, d)$ and $H_0(t, d)$ for the parameters $d = 3, 14, 25, 36, 47, 58, 69, 80, 91$. This is represented in Figs 12 and 13. Your students should carefully inspect and compare the 3-D and the 2-D graphs!

With increasing parameter $d$, the curves shift their maximum from right to left and become broader. The discussion of the families of curves makes it easier to understand the integral terms $H_w(t, d)$ and $H_0(t, d)$. In both cases, the global maximum is achieved at $d > 36\,\text{ms}$. It follows that the maximum for $d = 36\,\text{ms}$ defines the HIC; the parameter curves show the respective values. Table II demonstrates that our model results correspond very well with the Mercedes-Benz measurements.
Critical review of the model

The complicated formula, the huge computer used for the calculation and the exact numerical result feign a meaningfulness that crash-modelling using the HIC does not have. Reality was simplified very strongly in the modelling. Only the acceleration curve, measured with the help of dummies, has influenced the mathematical model. Probably, the formula defining the HIC was created by accident: someone tried to improve the SI, got more or less satisfactory results, and this formulation became the general norm. The power of 2.5 means that doubling the deceleration produces the same result as multiplying the relevant time of the deceleration by the factor 5.7. Such things "smell" like a rule of thumb: a power of 2 means a factor 4 for the time, which is perhaps a
Experts agree that $HIC$ values above 1000 are absolutely life threatening. With respect to the various incalculabilities of crash tests (and, as I suspect, in order not to annoy producers who do not reach acceptable values) one adds an addition of 25 per cent. and speaks of a low injury risk up to an $HIC$ value of 1250, of a medium rise from 1251 to 1500, and of a high injury risk only for values above 1500. Similarly, one estimates the result of the maximal, at least 3 ms duration, head load as being low up to 60g, medium from 61 to 70g, and high only above 70g. Indeed, the range of variation of the $HIC$ is very large as Figs 14–17 from different crash tests show (one can see such measurement curves in the motor press). The first three cars did not have an airbag. The Citroen and Fiat cars were tested in 1992, the Golf was tested in 1991. The Mercedes tested in 1993 was equipped with an airbag and the influence of the airbag is obvious! It also becomes clear how the $HIC$ value can be reduced solely by use of construction features. The given deceleration values are the maximal values for at least 3 ms duration.

Fig. 13. Family of curves with parameter $d$ of $H_0(t,d)$ for the crash without airbag.

Fig. 14. Citroen ZX: $HIC = 1368, a = 110g.$

Fig. 15. Fiat Tipo: $HIC = 818, a = 83g.$

Fig. 16. VW Golf III: $HIC = 455, a = 60g.$
Today, modern construction reaches better values. For example, a crash test of the Audi 8 (in summer 1995, with airbag) yielded an $HIC$ value of 142 and an A-3ms value of 33g. Unfortunately, such good values cannot be reached by small cars so far.

Just as with students’ examination marks, the $HIC$ does not provide an interval scale that enables comparisons. The $HIC$ cannot make any statement about kind and severity of eventual injuries but it gives an initial orientation for an estimation of the general injury risk.

References
3. SAE Information Report J 885, pp. 220–222 (with thanks to the car development division of the Mercedes-Benz AG).

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