ANALYSIS OF INSTANTANEOUS ENERGY
OF NORMAL GAIT

DAVID A. WINTER
Department of Kinesiology, University of Waterloo, Waterloo, Ontario, Canada

and

ARTHUR O. QUANBURY and GARY D. REIMER
Shriners Hospital for Crippled Children, Winnipeg, Manitoba R3M OA8, Canada

Abstract—An analysis of the energy components of human gait in the sagittal plane has been made employing kinematic and anthropometric data. The television data source does not encourage natural gait and is sufficiently complete so that drastic simplifying assumptions are not necessary for the analysis. The instantaneous kinetic (translational and rotational) and potential energy is calculated for the shank, thigh and torso. The following conclusions can be drawn:

(i) The torso acts as a conservative system with about half of its change in potential and kinetic energy interchanging.

(ii) The thigh conserves ca. 1/3 of its energy change.

(iii) The shank has the largest total energy changes, but virtually no exchange between kinetic and potential.

(iv) In all but the shank the rotational kinetic component, \( \frac{1}{2} I \omega^2 \), can be ignored.

(v) The stride-to-stride changes in total body energy were less than 4\%, of the average body energy.

INTRODUCTION

A large number of statements have been made regarding the ability of the human body to optimize its energy consumption. Inman (1966) summarized this feeling in a hypothesis "that the body will integrate the motion of various segments of the body and control the activity of muscles so that the energy required by each step is minimal". Work physiology studies (Bresler, 1951; Ewald, 1961) give strong indication that the natural cadence is close to that required for minimum energy expenditure. Nubar and Contini (1961) in a biped model of human gait, illustrated a mathematical approach in the determination of minimum energy during static conditions. However, the instantaneous energy of each of the body segments has not been analysed in complete detail, mainly because appropriate data has not been available for computer analysis. Most researchers have been forced to make compromising assumptions. Townsend and Seireg (1977), Beckett and Chang (1968), Bresler et al. (1957) assume sinusoidal trajectories in their analysis. In ignoring the higher harmonics they introduce errors when they calculate the velocities necessary for their analysis. The higher harmonics are not insignificant, especially in the analysis of the energy of the shank, which, it will be seen, is the limb component which has the highest energy changes during the gait cycle. Other researchers (Cavagna et al., 1971; Gerten et al., 1969) have assumed that the energy change of the anatomical centre of gravity alone represents the energy changes of the body as a whole. In an earlier paper Cavagna et al. (1963) made corrections for the movement of the centre of gravity as a result of limb movements. Major corrections were made for vertical displacements (potential energy changes) and horizontal velocity (translational kinetic energy) of the limbs. However, the kinetic energy associated with vertical velocity and rotation of the limb segments, although of second order importance, was not considered. Within these limitations the exchange of energy at the corrected centre of gravity was determined and further expanded in an excellent paper (Cavagna and Margaria, 1966) in which the energy changes were related to walking speed.

The basis of this paper is to calculate directly the energy of each body segment in the sagittal plane without any drastic simplifying assumptions, and to show the energy exchanges within segments, and their individual contributions to the total energy changes during the walking cycle.

INSTANTANEOUS ENERGY ANALYSIS

Energy can be stored in two forms: kinetic and potential energy. The kinetic energy has two components: translational energy, in which the energy is stored as a result of the linear velocity of its centre of mass, and rotational energy, where (like a propeller) the energy is a function of its angular velocity in a given plane. Beckett and Chang (1968), Townsend and Seireg (1972), Bresler et al. (1957) and Ralston and Lukin (1969) have recognized these components but all have made simplifying assumptions that introduce varying degrees of error in the calculated energies.
Consider the ith segment. the instantaneous energy $E_i$ is:

$$E_i = m_i g h_i + \frac{1}{2} m_i v_i^2 + \frac{1}{2} I_i \omega_i^2,$$

where $m_i$ is the mass of the segment, $h_i$ is the height of the centre of mass above a reference datum, $v_i$ is the linear velocity of the centre of mass, $I_i$ is the rotational moment of inertia about the centre of mass and $\omega_i$ is the angular velocity of the ith segment. It should be noted that $v_i$ and $\omega_i$ must be absolute velocities in space. In the data to be presented the analysis is confined to the sagittal plane. The energy changes in the transverse plane (medial-lateral) are assumed to be negligible (Cavagna et al., 1963).

**SOURCE OF KINEMATIC DATA**

All data is collected and converted by means of a TV-computer system (Winter et al., 1972) which yields the trajectory (in absolute coordinates) of anatomical markers. The location of these markers is shown in Fig. 1, along with the convention for limb segment and joint angles. From this coordinate data the angles can be readily calculated, along with the desired velocities and accelerations (Winter et al., 1974a; Winter et al., 1974b).

For example, the angular velocity of the shank is calculated from the first time derivative of the shank angle, $\theta_{sh}$. The location of the centre of mass of each limb segment is determined from anthropometric data provided by Contini (1971) and by relating this data to the location of our markers. For example, for the centre of mass of the shank:

$$x_{sh} = x_4 + a(x_3 - x_4),$$

$$y_{sh} = y_4 + a(y_3 - y_4),$$

where $a$ is the fraction of the distance that the centre of mass is located between marker 3 and 4. Thus

$$\text{Thigh Angle} = \theta_{31},$$

$$\text{Shank Angle} = \theta_{43},$$

$$\theta_{sh} = \theta_{31} - \theta_{43},$$

$\theta_{sh}$ is the sum of the angle and its time derivatives increase in a counter clockwise direction.

The magnitude of the shank velocity, $v_{sh}$, and the direction of the velocity vector, $\theta_{sh}$, can be calculated from the relationship:

$$v_{sh} = \sqrt{x_{sh}^2 + y_{sh}^2}$$

and

$$\theta_{sh} = \tan^{-1}\left(\frac{y_{sh}}{x_{sh}}\right).$$

A plot of $v_{sh}$ during one walking stride along with $\theta_{sh}$ is plotted in Fig. 2. Although $\theta_{sh}$ does not enter into the calculations of kinetic energy, it is presented here to indicate that errors will be introduced when the velocity in the forward direction is considered rather than the magnitude of the velocity vector. The magnitude of the masses and rotational moments of inertia are also calculated from tables supplied by Contini. Therefore the three components of energy of the shank and thigh can be readily calculated for each point in time (every TV field, or 60 points/sec).

For the energy of the torso certain simplifying assumptions were made. Data was obtained from the right leg only. Because of the repetitive nature of normal gait we assumed that the trajectory of the left leg was the same as that of the right leg, displaced in time by half a stride period. Thus the right leg data, suitably displaced in time, became data for the left leg. To determine the trajectory of the centre of mass of the trunk we assumed it was the same as the average of the trajectories of marker No. 1 (greater trochanter) for the left and right leg. Thus the potential and kinetic translational energy of the
Data was analyzed for 3 or more strides on each of 5 normal subjects. The pattern of energy changes was very similar for all subjects; the analysis presented here is representative of the group. A summary of the energy changes of the 5 subjects is tabulated in Table 1 which appears later in this paper.

Figures 3, 4 and 5 present the instantaneous energy of the shank, thigh and trunk respectively, for swing phase. The translational, rotational and potential energy components are also plotted. It should be noted that the datum height for zero potential energy was floor level. Thus the ordinate axis may have a broken scale in order to show the mean range of potential energy, and at the same time have a reasonable scale to show the detailed energy changes during the walking cycle. We emphasize that the energy changes, rather than the absolute energy level, are important because an energy change indicates power flow to or from the limb segment in question. In Fig. 6 a plot is made of the entire stride, showing the energy of the left and right legs compared with the torso energy, and the total energy of the body.

DISCUSSION

1. Shank energy

From Fig. 3 it can be seen that the change in kinetic energy (ΔKE) is ca. 14 J and is considerably larger than the change in potential energy (ΔPE), which was only 3 J. The minimum potential energy was 17 J and as expected, this occurs during stance phase. The large difference between ΔKE and ΔPE rules out any
possibility of a reasonable passive energy exchange. Also, both kinetic and potential energy components are seen to be increasing at the same time, during the first half of swing, and decreasing during the latter half of swing. These changes in energy must have come from muscle activity at the knee joint and via translational energy transferred to the shank due to the movement of the knee joint itself. The rotational kinetic component is small compared with the translational component, but not too small to be ignored, especially in higher speed walking or running (Winter 1974b).

### ii. Thigh energy

From the results plotted in Fig. 4 there appears to be a noticeable energy exchange within the limb itself. \( \Delta KE \) is due almost entirely to translation, and is ca. 9 J, while \( \Delta PE \) is about 4 J, and is a small fraction of its mean potential energy (49 J). The first energy exchange of energy takes place between 100 and 300 msec of swing. Here the kinetic energy decreases while the potential energy increases as the thigh is lifted off the ground prior to swing through. Similarly, during the latter part of swing some of this stored potential energy is converted back into kinetic energy. Conversion of energy reverses again during stance when the thigh rises to its maximum height while it reaches its lowest velocity. The net result of these energy interchanges is that the total peak-to-peak change in energy of the thigh is 6 J, which is 3 J less than \( \Delta KE \) component itself.

### iii. Comparison of thigh and shank energy

A comparison of the energy changes in the thigh vs the shank indicates that the shank requires 16 joules vs 6 joules for the thigh. Bresler et al. (1957) calculate the shank energy change to be ca. 29 vs 15 J for the thigh. Ralston and Lukin (1969) ignored any contribution from vertical velocities in their kinetic component, and calculated the shank energy to be 18 vs 13 J for the thigh. The higher values calculated by these two research groups may be attributed to the use of treadmill, a higher cadence and a large number of attachments.

### iv. Torso energy

From Fig. 5 the energy levels are quite high due to the mean levels of potential energy (395 J) and kinetic energy (35 J). \( \Delta KE \) and \( \Delta PE \) are almost equal in amplitude (15–17 J) and are almost 180° out of phase, indicative of a distinct interchange of energy. During the middle of each stance phase the torso rises to its maximum height, and at the same time its forward velocity is slowed to a minimum. At the end of single support the torso falls forward and drops, giving up some of its potential energy to kinetic energy which reaches a maximum during double support phase. The net result of these energy interchanges is that the total energy change of the torso is about 10 joules, which is somewhat less than \( \Delta KE \) or \( \Delta PE \). Similar interchanges of energy in the trunk were observed by Ralston and Lukin (1969).

### v. Total energy

From the total energy curve (Fig. 6) we note that its major increase occurs around heel contact of each leg. Some of this initial increase occurs prior to toe off when push-off forces add to the torso kinetic energy. However, the major cause of the rise in the total energy can be attributed to the lift off of the swing phase leg resulting in increased kinetic and potential components of both the thigh and shank. Ralston and Lukin (1969) have also noted this rapid energy change but attribute most of it to "push-off" energy, and the balance to transfer of energy from the decelerating leg to the torso (immediately prior to heel strike). However, it can be seen from Fig. 6 that the sharp drop in energy due to the decelerating leg does not coincide with an increase in torso energy.

The energy exchange at the centre of gravity, as calculated by Cavagna (1963), was somewhat similar in shape and magnitude to the energy changes we found within the torso, and the total energy curve is also somewhat similar. He calculated two energy components—potential energy changes in the centre of gravity (which he called gravitational work) and work due to horizontal velocity changes (which he called forward work). An exact comparison cannot be made because our total energy is a sum of segment
energies rather than types of energy. However peak-to-peak total energy is ca. 4 cal (16.7 J). This compares with our peak-to-peak energy change of ca. 22 J. It is also noted that the mean body energy is almost 600 J, so this stride-to-stride change of 22 J represents a change of less than 4\% of the average energy of the body as a whole. The mean body energy is dependent on the zero energy datum chosen for the potential energy component, which is the floor level in this analysis. A different datum would result in a different mean body energy and hence a different per cent change in total energy from stride-to-stride. These changes occur twice per stride so it appears that two bursts of 4\% of the total energy must be added each stride, and two similar decelerating bursts of energy are also required per stride. It is anticipated that the number and magnitude of these $\Delta E$ changes will be a measure of the efficiency of walking.

**Table I.**

<table>
<thead>
<tr>
<th>Subject</th>
<th>Sex</th>
<th>Weight (kg)</th>
<th>Walking speed (m/sec)</th>
<th>Shank energy (J)</th>
<th>Thigh energy (J)</th>
<th>Body energy, $\Delta E$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>F</td>
<td>55.7</td>
<td>1 3 1 2 1</td>
<td>$\Delta P E$</td>
<td>$\Delta K E$</td>
<td>16 12 16 22 14</td>
</tr>
<tr>
<td>2</td>
<td>M</td>
<td>58.9</td>
<td>1.3 1.2 1.1 1.5 1.0</td>
<td>3 3 3 3 3</td>
<td>10 13 20 11</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>M</td>
<td>58.9</td>
<td>1 3 1 2 1</td>
<td>3 3 3 3 3</td>
<td>10 13 20 11</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>F</td>
<td>55.7</td>
<td>1 3 1 2 1</td>
<td>3 3 3 3 3</td>
<td>10 13 20 11</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>M</td>
<td>68.6</td>
<td>1 3 1 2 1</td>
<td>3 3 3 3 3</td>
<td>10 13 20 11</td>
<td></td>
</tr>
</tbody>
</table>

| Mean    | 585 730 740 600 840 |

**CONCLUSIONS**

The individual component of energy of each segment of the human body has been calculated for normal walking. The following conclusions can be made:

(i) The torso acts as a conservative system with about half of its change in kinetic and potential energy interchanging.
(ii) The thigh also conserves ca. 1/3 of its energy change.
(iii) The shank has the largest increase in total energy, but virtually no exchange.
(iv) In all but the shank the rotational kinetic component, $I \omega^2$, can be ignored. In the shank it contributes ca. 10\% of the shank energy but will be more significant at fast speeds and in running.
(v) The stride-to-stride changes in body energy were less than 4\% of the average body energy.

A complete analysis assessment of the energy flow from one segment to another requires a more detailed analysis, including the calculation or measurement of joint moments and joint forces (Elfman, 1939). The measurement of joint forces requires an accurate force plate, which is not available at this time in our laboratory.

**Acknowledgement**—The authors wish to acknowledge the continued support of the Shriners Hospital (Winnipeg Unit) and the Medical Research Council of Canada (Grant MT 4343).

**REFERENCES**


