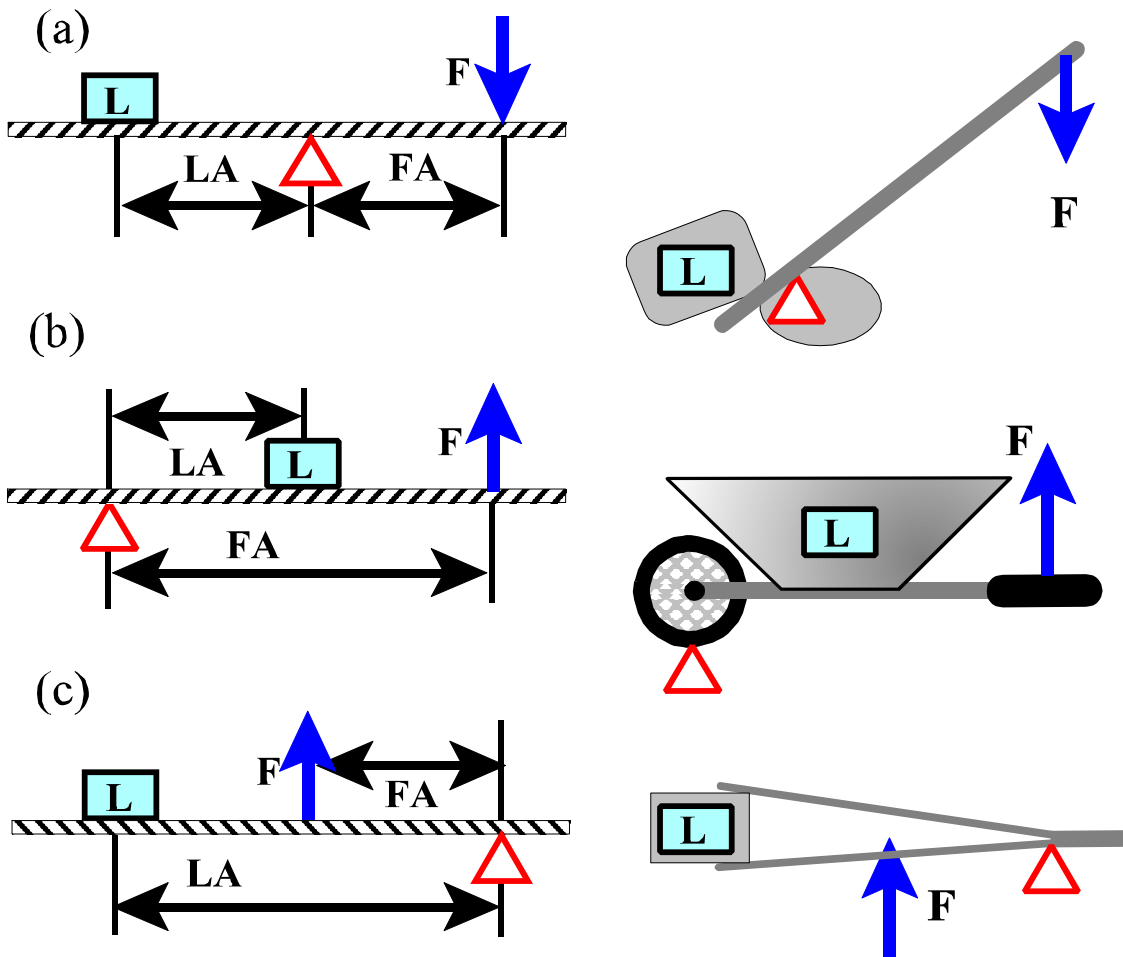


Levers of the Musculoskeletal System

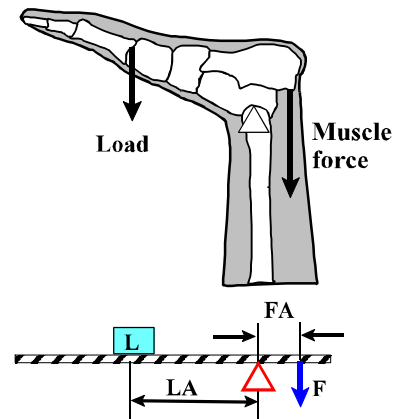
- Lever system consists of:
 - lever
 - fulcrum
 - load
 - force
- Three classes of levers
 1. first class (a) - pry bars, crowbars
 2. second class (b) - wheelbarrow
 3. third class (c) - tweezers, most muscles



Levers of the Musculoskeletal System

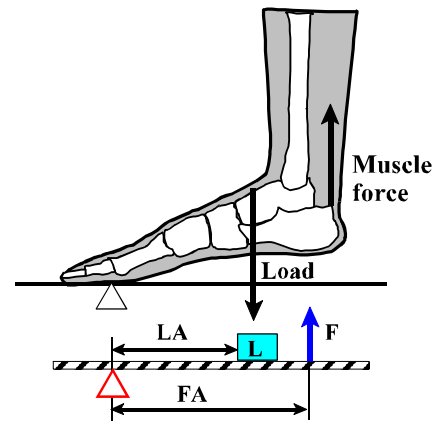
First class lever.

Gastrocnemius inverted



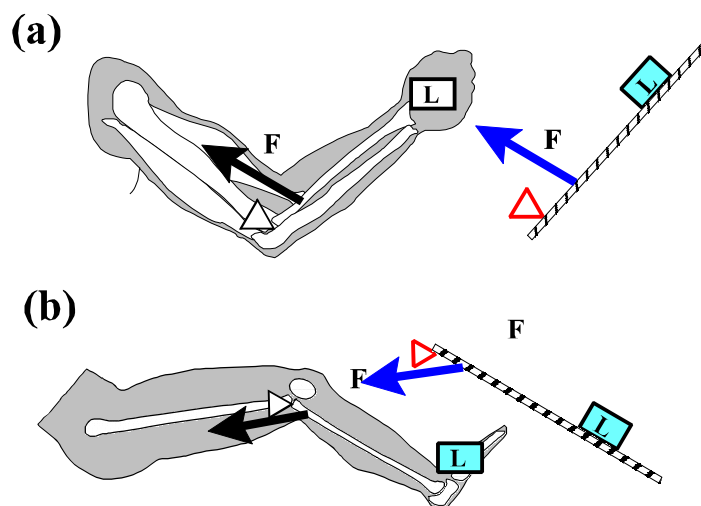
Second class lever.

Gastrocnemius during standing



Third class lever.

Biceps brachii and hamstring muscles



Moments of Force

Moment of a Force:

- turning effect of a force
- also called torque (especially when effect is about the longitudinal axis)
- product of moment arm times force

$$M = F d \quad (F \text{ is perpendicular to } d)$$

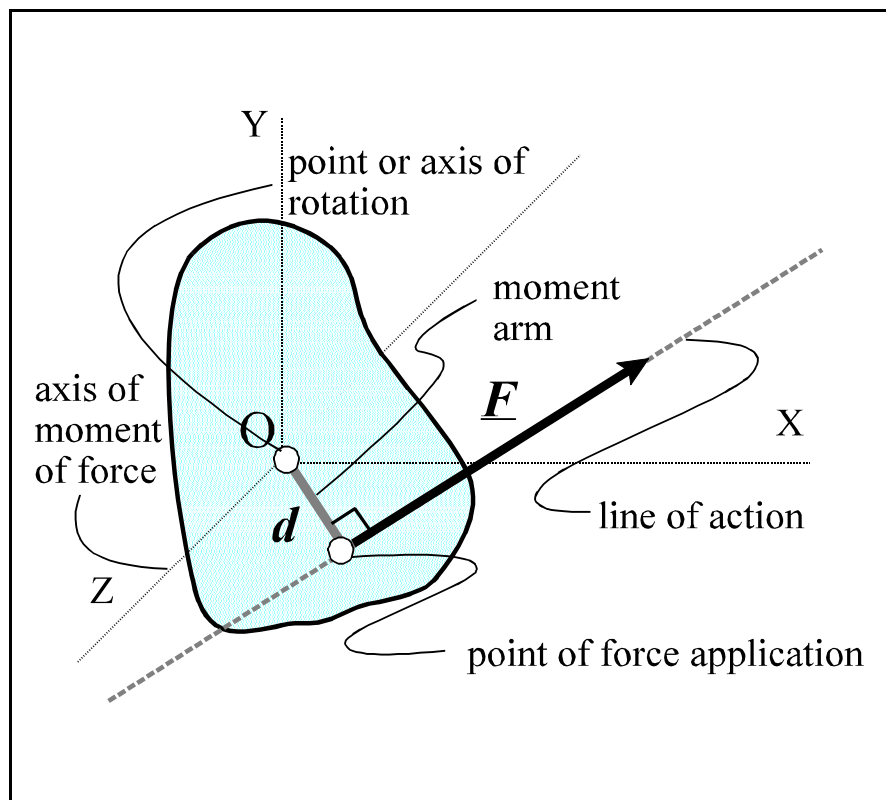
or

$$M = r F \sin \theta \quad (\theta \text{ is angle between } F \text{ and } r)$$

- units are newton metres or N.m

Moment: - is the perpendicular distance between a line (e.g., line of force) and a point or an axis. (other uses: moment of inertia, moment of momentum)

Direction: - counter-clockwise (right-hand rule) is positive, clockwise is negative



Moment of a Force

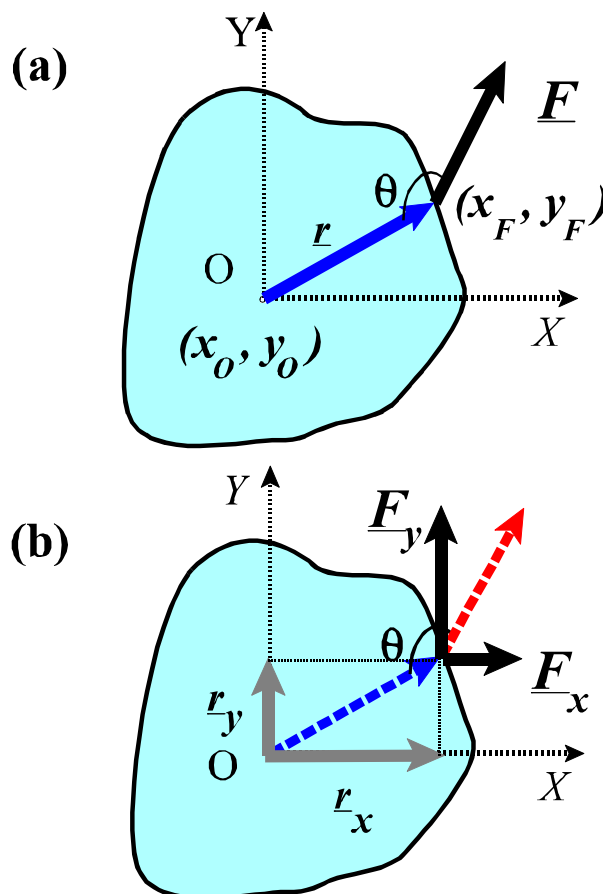
- vector product of position vector (\underline{r}) and force vector (\underline{F})

$$\underline{M} = \underline{r} \times \underline{F} \quad (\text{moment vector})$$

$$= M_z \underline{k} \quad (\text{using unit vector } \underline{k})$$

$$= (r_x F_y - r_y F_x) \underline{k} \quad (\text{expansion})$$

$$[\underline{M}]_z = M_z = (r_x F_y - r_y F_x) \quad (\text{scalar moment about the Z axis})$$



Example: $\underline{r} = (26, 15) \text{ cm}$ $\underline{F} = (300, 450) \text{ N}$

$$M_z = (26 \times 450) - (15 \times 300) = 7200 \text{ (N.cm)} = 72.0 \text{ (N.m)}$$

Statics

Statics:

- field of applied mechanics concerned with rigid bodies that are in equilibrium
- mechanics of rigid bodies at rest or in constant **linear** motion, also called static equilibrium
- mainly used in the analysis of structures or the stability of bodies (e.g., starting positions)

Laws of Statics

- derived from Newton's First Law:

$$\Sigma \underline{F} = \underline{\mathbf{0}} \quad \text{or for 2D motions}$$

$$\Sigma F_x = 0 \quad \text{and} \quad \Sigma F_y = 0$$

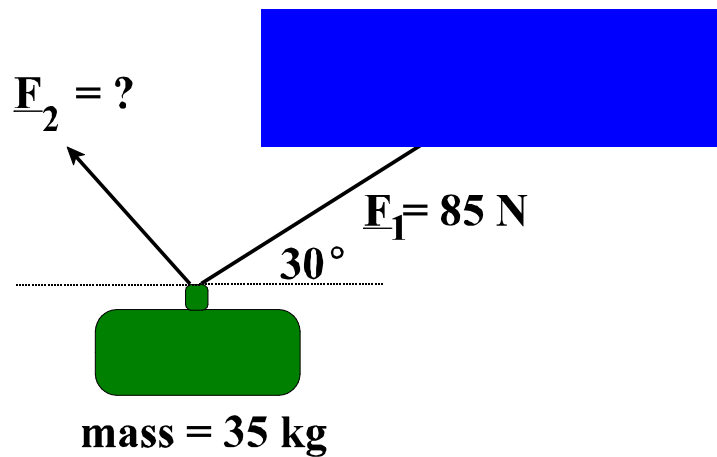
and

$$\Sigma (\underline{r} \times \underline{F}) = \underline{\mathbf{0}} \quad \text{or for 2D motions}$$

$$\Sigma M_z = 0$$

- thus, in 2D, there are three independent equations ($\Sigma F_x = 0$, $\Sigma F_y = 0$ and $\Sigma M_z = 0$)
- in 3D there are six equations (3 for forces, 3 for moments)
- i.e., sum of all forces produces a closed polygon and sum of moments equal zero

Example:



$$\Sigma F_x = 0: \quad F_{1_x} + F_{2_x} = 0$$

$$85 \cos 30^\circ + F_{2_x} = 0$$

$$F_{2_x} = -85 \cos 30^\circ = -73.6$$

$$\Sigma F_y = 0: \quad F_{1_y} + F_{2_y} - m g = 0$$

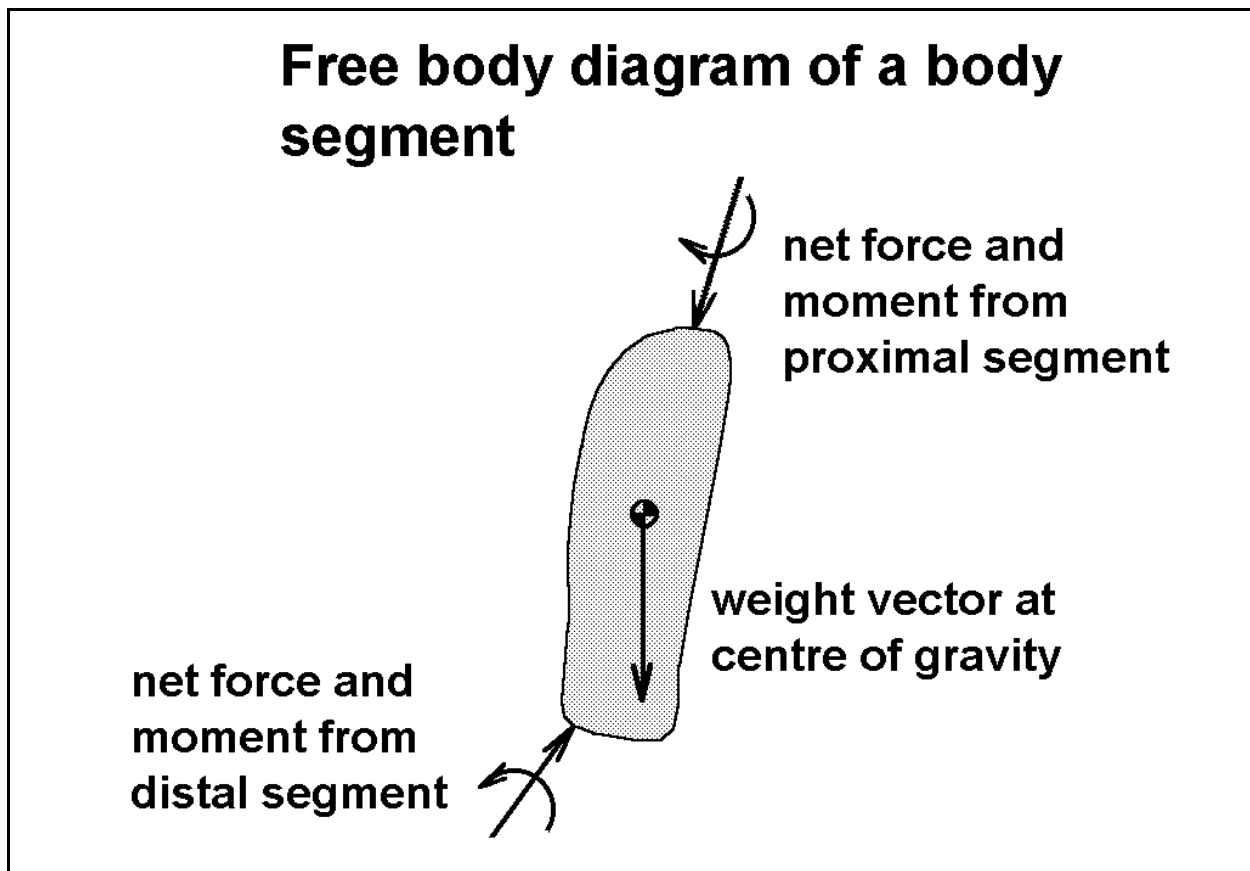
$$85 \sin 30^\circ + F_{2_y} - (35 \times 9.81) = 0$$

$$F_{2_y} = 343 - 85 \sin 30^\circ = 301$$

Free-Body Diagrams

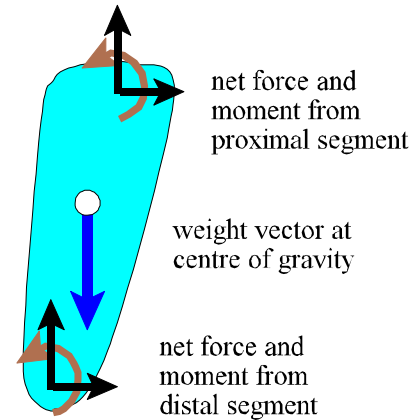
Rules:

1. **Select** the appropriate body or body segment, keeping in mind the results you want to achieve.
2. Draw all **known** forces or moments of force at their respective points of application. (E.g., weight vector, measured external forces, etc.)
3. Draw all **unknown** forces and moments that directly influence the free body, especially at points of detachment from the environment or other parts of the body. (E.g., at severed joints, ground if no force platform, etc.)
4. Do NOT include **internal** forces which originate and terminate within the free body. (E.g., muscle or joint forces of imbedded joints)

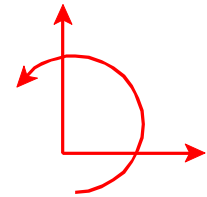


Steps for Solving Mechanics Problems

Step 1. Draw a **free-body diagram** (FBD) of the body (or bodies) of interest.



Step 2. Select and draw an appropriate **axis system** that defines the positive directions and slopes of the axes (e.g., X-Y, polar, normal-tangential, radial-transverse, see figure 3.32). Usually a right-handed axis is selected.



Step 3. For each free-body diagram write out the **equations of motion** or **principles** that apply to the problem. These are based upon the six fundamental laws or their corollaries.

$$\begin{aligned} \text{E.g., } \Sigma F_x &= 0 \\ \Sigma F_y &= 0 \\ \Sigma M_z &= 0 \end{aligned}$$

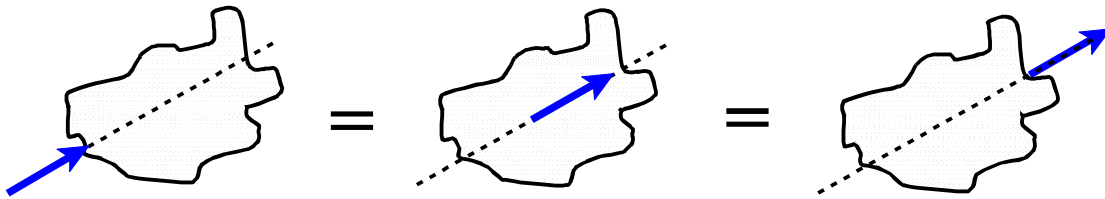
Step 4. **Solve the equations** for the unknowns (Note, this can only be done if the number of unknowns is less than or equal to the number of equations).

Step 5. **Write out the solutions** with the directions and signs (+/-) based on the axes selected in step 2 and with appropriate accuracies and SI units.

Step 6. **Check** your calculations by recomputing the solutions.

Principle of Transmissibility

One of the six fundamental principles of mechanics. A force acting on a rigid body will produce the same effects no matter where it is applied as long as it is in the same direction and same line of action. We can call such forces, **sliding vectors**, because they can slide up or down their lines of action without affecting their influence on the rigid body.



Force vectors can slide along their lines of action.

push = pull



Deformable bodies do NOT follow transmissibility principle.

