Projectile Motion

- occurs when an object has only **gravitational force** acting on it
- forces such as **air resistance** are assumed to be negligible
- object may be considered a **particle** at location of centre of gravity
- FBD is a point with weight vector as only force
- usually can only be used for short falls/flights where velocity is low
- in true “free-fall” situations, as velocity increases so does air resistance until “terminal velocity” is reached
- **terminal velocity** occurs when weight vector equals drag force of air resistance at which point the object will fall at constant (terminal) velocity
- with projectile motions acceleration of object (near earth) is 9.81 m/s^2 downwards. Acceleration is zero, meaning constant velocity, in any horizontal direction
- therefore equations for **constant linear acceleration** (vertically) and **constant linear velocity** (horizontally) apply

Projectile Motion Equations

$$s_{f_x} = s_{i_x} + v_{i_x} t \qquad (1)$$

$$v_{f_y} = v_{i_y} - gt \qquad (2)$$

$$v_{f_y}^2 = v_{i_y}^2 - 2g(s_{f_y} - s_{i_y}) \qquad (3)$$

$$s_{f_y} = s_{i_y} + v_{i_y} t - 1/2gt^2 \qquad (4)$$

$$s_{f_y} = s_{i_y} + 1/2(v_{i_y} + v_{f_y})t \qquad (5)$$

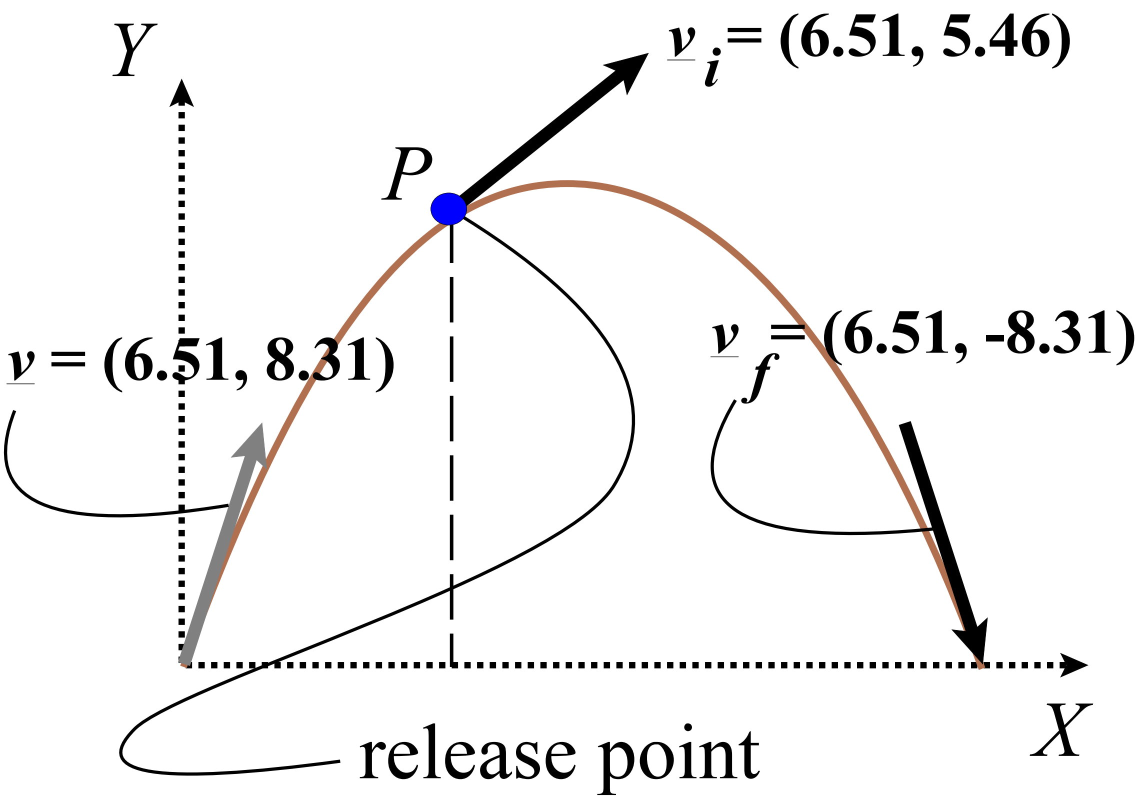
Where:

s_i = initial position
s_f = final position
v_i = initial velocity
v_f = final velocity
g = acceleration due to gravity = 9.81 m/s^2
t = duration in seconds

Trajectory of a Projectile

Example:

Ball is released from point, P (0.0, 2.00), with a velocity of (6.51, 5.46) metres per second. Where will it land?



Solution:

Cannot find horizontal distance without knowing flight time therefore use vertical motion equations to solve for the time then apply horizontal motion equation to get landing distance.

Solution

Equation 4 (previous page) could be used but you would have to "find the roots of a quadratic equation." This is a difficult problem (unless the initial velocity is zero). An alternate approach is to solve for the final velocity with equation 3, then apply equation 2 to compute the time.

$$\begin{aligned} v_{f_y}^{\;2} &= v_{i_y}^{\;2} - 2g(s_{f_y} - s_{i_y}) \\ &= 5.464^2 \; -2(9.81)(0-2.0) \\ &= 29.86 + 39.24 = 69.1 \\ v_{f_y} &= \pm\sqrt{69.10} = \pm 8.31 \quad [\text{m/s}] \end{aligned}$$

Because of the square root operation there are two solutions to this equation. One solution is "extraneous." It represents the velocity "before" the projection started. Thus, we ignore the positive (upwards) velocity and choose the negative velocity as the correct answer.

Now, compute the flight time.

$$\begin{aligned} t &= \frac{v_{f_y} - v_{i_y}}{-g} \\ &= \frac{-8.31 \;\; -5.46}{-9.81} = \frac{13.77}{9.81} \\ &= 1.404 \quad [\text{s}] \end{aligned}$$

Finally, use the equation for constant speed horizontal motion to compute the landing distance.

$$\begin{aligned} s_{f_x} &= s_{i_x} + v_{i_x} t \\ &= 0 + 6.51 \times 1.404 = 9.14 \quad [\text{m}] \end{aligned}$$

Thus, the person will travel 9.14 metres forward.