Angular Kinematics

Average Angular Velocity

$$\bar{\omega} = \frac{\theta_{final} - \theta_{initial}}{time}$$

Instantaneous Angular Velocity

$$\omega = \frac{d\theta}{dt}$$

Average Angular Acceleration

$$\bar{\alpha} = \frac{\omega_{final} - \omega_{initial}}{time}$$

Instantaneous Angular Acceleration

$$\alpha = \frac{d\omega}{dt}$$

Constant Angular Acceleration Equations

$$ {}_{f} = \quad {}_{i} + \quad t $$

$$ {}_{f}^{2} = \quad {}_{i}^{2} + \qquad (\quad {}_{f} - \quad {}_{i}) $$

$$ {}_{f} = \quad {}_{i} + \quad {}_{i} t + ½ \quad t^2 $$

$$ {}_{f} = \quad {}_{i} + ½ (\quad {}_{i} + \quad {}_{f}) $$

Where:

$_{i}$ = initial angular position

$_{f}$ = final angular position

$_{i}$ = initial angular velocity

$_{f}$ = final angular velocity

= angular acceleration

t = duration in seconds

Example: How many revolutions will a person complete after initiating a spin at 5 revolutions per second and accelerating at the rate of 2 r/s^2 for 3 seconds?

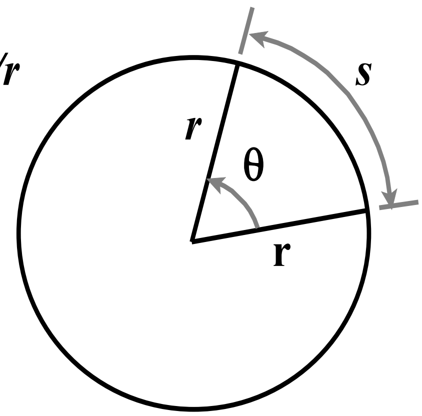
$_{f}$ = 0 + 5 (3) + ½ (2) 3^2 = 15 + 9 = 24.0 [revolutions]

Note, keep all angular units within an equation in the same units, i.e., degrees (deg), radians(rad) or revolutions (r).

Angular Kinematics

Radian Measure:

$\theta = s/r$

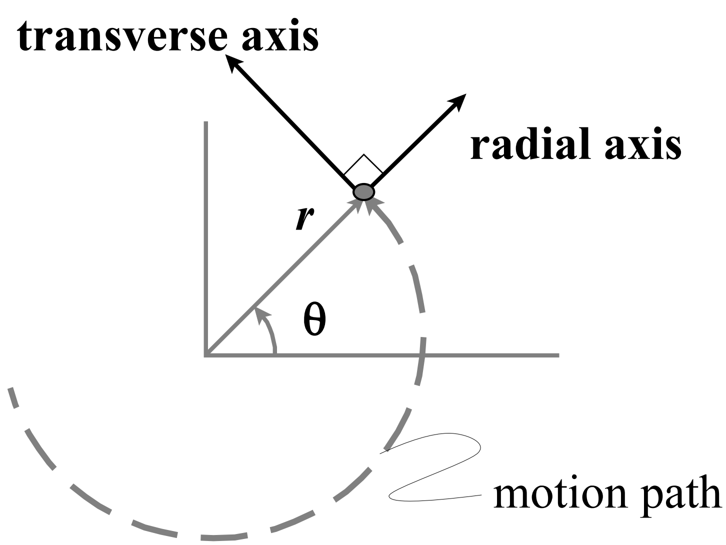


When $s = r$, $\theta = 1$ radian

r **= radius**
s **= arc length**
= angle

Since, $= s / r$
therefore, $s = r$, where is in radians

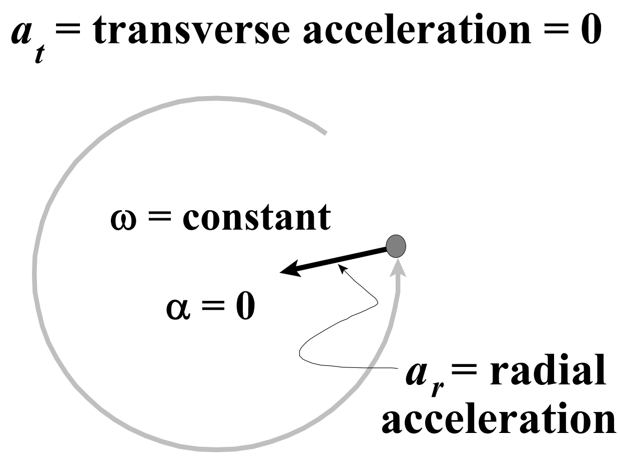
Radial and Transverse Axes:



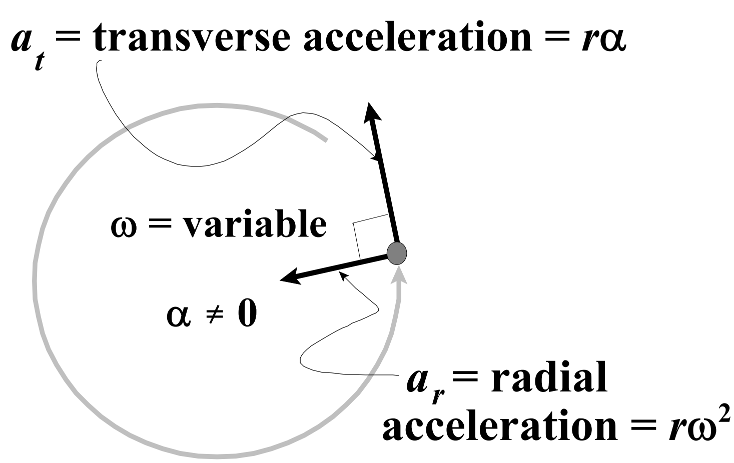
- **radial axis** begins at the point and is directed along the line from the origin to the point
- **transverse axis** is orthogonal to radial (i.e., +90 degrees rotation)

Angular Kinematics

Constant Angular Velocity (= 0):



Variable Angular Velocity (≠ 0):



Relationships between Linear and Angular Kinematics

For Circular Motion:

$= s_{transverse} / r$ or

$s_{transverse} = r$ and

$s_{radial} = r$

Furthermore, $v_{tranverse} = r$

$v_{radial} = 0$ (since r is constant)

and $a_{transverse} = r$

($a_{transverse}$ occurs when angular speed changes)

$a_{radial} = v_{transverse}^{2} / r = r \quad ^{2}$

(a_{radial} is due to directional changes)

$a_{total} = \sqrt{(a_{radial}^{2} + a_{transverse}^{2}}$

(a_{total} is the magnitude of the resultant acceleration)

Example:

A bucket is swung in a circular path with an angular acceleration of 20.0 rad/s^2 at a radius of 1.250 cm. What is the linear velocity and acceleration when the bucket reaches an angular velocity of 25.0 rad/s?

$v_{transverse} = r \quad = 1.250\ \text{m} \times 25.0\ \text{rad/s} = 31.25\ \text{m/s}$
$v_{radial} = 0.0$

Thus, the linear velocity vector is: $\underline{\boldsymbol{v}} = (0.00,\ 31.3)$ m/s

$a_{transverse} = r \quad = 1.250\ \text{m} \times 20.0\ \text{rad/s}^2 = 25.0\ \text{m/s}^2$
$a_{radial} = r \quad ^2 = 1.250\ \text{m} \times (25.0\ \text{rad/s})^2 = 781.25\ \text{m/s}^2$
or $a_{radial} = v_{transverse}{}^2 / r = 31.25^2 / 1.250 = 781.25\ \text{m/s}^2$

Thus, the linear acceleration vector is: $\underline{\boldsymbol{a}} = (-781,\ 25.0)$ m/s^2